A New, and moſt uſeful

INSTRUMENT

FOR

ADDITION

AND

SUBSTRACTION

OF

Pounds, Shillings, Pence,

and *Farthings*;

Without charging the Memory, diſturbing the Mind, or expoſing the Operator to any uncertainty; Which no Method hitherto publiſhed, can juſtly pretend to.

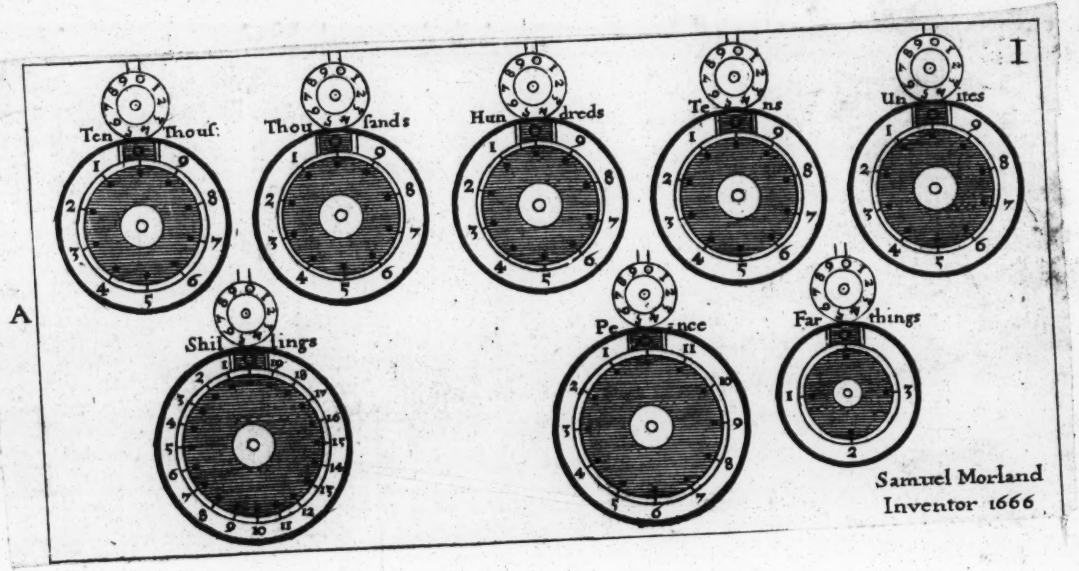
BIBLIOTHECA
COLL: SS: TRIN:
DUBLIN:

Invented and Preſented to His moſt Excellent Majeſty *CHARLES* II. King of *Great Britain, France,* and *Ireland,* &c. 1666.

By *S. MORLAND.*

And by the importunity of his very good friends, made publick 1672.

L. p. 56


I
A
Ten Thousf:
Thou sands
Hun dreds
Te ns
Un ites
Shil lings
Pe nce
Far things
Samuel Morland
Inventor 1666

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DUBLIN.

TO

Set or Diſpoſe

THE

INSTRUMENT

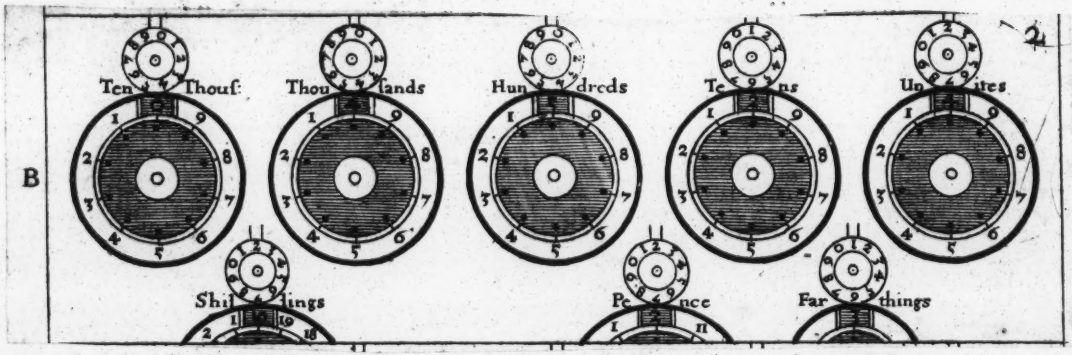
FOR

OPERATION:

Which Inſtrument *muſt be held in the Left hand, and the* Index *or blew* Pin *in the Right hand, as men uſually hold a Pen, but ſomething more upright or perpendicular.*

IN thoſe five upper Plates, (in every of which there are 10 ſmall holes) if any of the *Digits* (ſuppoſe 3) appear in any of the *Windows*, put the point of the *Index* into the hole over againſt that Figure in the Margent, which is the Complement to 10. (namely 7.) And then turn it under the *Window*, where (0) will appear. This done, put the point of the *Index* into the (0) of the ſmall Plate above it; and then, if you would prepare the Inſtrument for *Addition*, turn about that ſmall Plate, till 0 ſtand juſt under that *Line* or mark in the upper Plate which is on the Right-hand, thus $\frac{11}{0}$ as in *Fig.* A. But if for *Subſtraction*, then turn 0 on under the Left-hand *line* or mark, thus $\frac{11}{0}$, as in *Fig.* D.

After the very ſame manner are all the lower plates of *Shillings*, *Pence*, and *Farthings* to be diſpoſed; Remembring onely that in the place of *Shillings*, where (20) is the number, the Gomplement to (3) is (17) And in *Pence*, the Complement to (3) is (9) And in *Farthings*, the Complement to (3) is (1) And


B
Ten Thousands
Thousands
Hundreds
Tens
Unites
Shillings
Pence
Farthings

thus are all the Plates fitted for Operation as in *Fig.* A.

This is likewiſe to be obſerved before the Inſtrument is ſet: If any of the Plates with Holes ſtick, and will not turn about forwards, it is becauſe neither (0) nor any *Figure* of the ſmall Plate above it is under the Right-hand-*line* or mark. And if they will not turn backwards, it is becauſe neither O, nor any *Figure* of the ſmall Plate above it is under the Left-hand-*line* or mark. And laſtly, if the ſmall Plate above will not move, it is becauſe no Hole in the lower Plate is directly under the *Window*.

The Operation of Addition.

Let the Sums to be added together, be theſe.

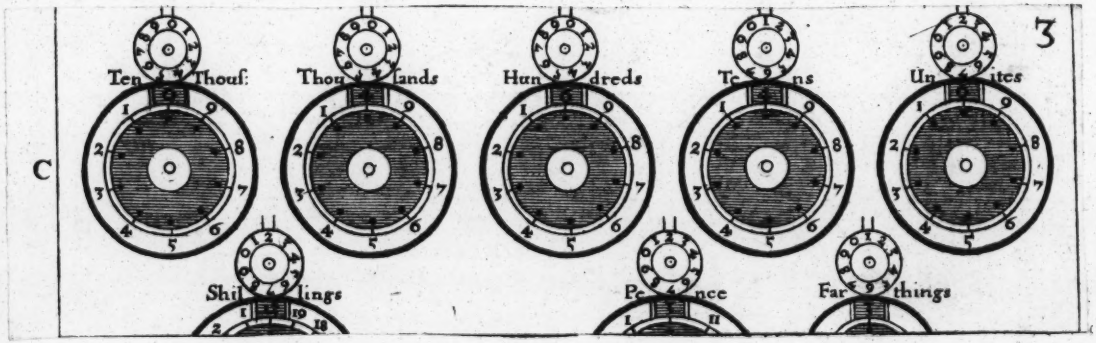
l.	*ſh.*	*d.*	*f.*
7	14	03	2
48	11	10	3
252	17	11	1
4337	8	2	1
4646	12	3	3

Firſt I ſet on 7 *l.* that is, I put the point of the *Index* into the hole of the place of *Unites* of Pounds, which is over againſt (7) in the Margent, and turn it under the *Window*. After that, I turn on (14) in the place of *Shillings*; (3) in the place of *Pence*; and (1) in the plaee of *Farthings*.

Again, as to the 2 Summe, I ſet on (48) thus, namely, (4) in the place of *Tens*, and (8) in the place of *Unites*: As likewiſe 11 in the place of *Shillings*; (10) in the place of *Fence*; and (1) in the place of *Farthings*. And having ſet on all the Summes in the manner and method aforeſaid, they will appear on the Inſtrument as in *Fig.* B.

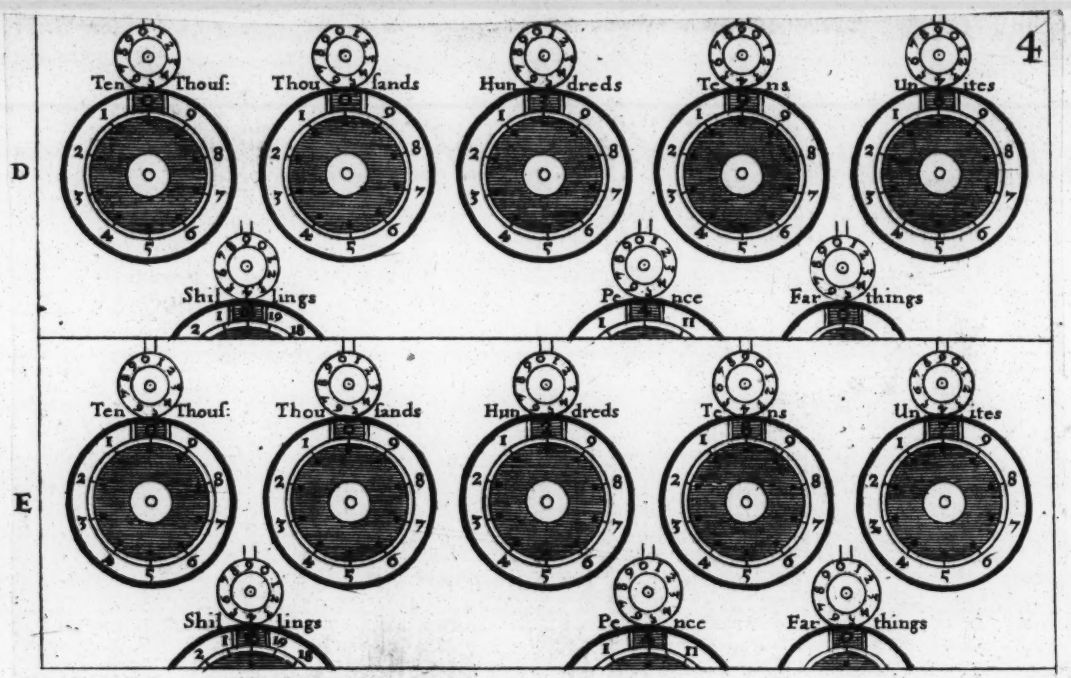
In fine, to perfect the Operation, I begin with the plaee of *Farthings*, and becauſe the *Figure* (1) of the

Plates


3
C
Ten Thousd:
Thou lands
Hun dreds
Te ns
Un ites
Shil lings
Pe nce
Far things

Plates over head ſtands between the two *Lines* or Marks, therefore I turn on (1) in the place of *Pence*. And becauſe (2) is between the two *Lines* in the place of *Pence*, I turn on (2) in the place of *Shillings*: And becauſe (2) is between the two *Lines* in the place of *Shillings*, I turn on (2) in the place of *Unites* of Pounds. Thus, for (2) between the two *Lines* in the place of *Unites* of Pounds, I turn on 2 in the place of *Tens*. And for (1) in the place of *Tens*, I turn on (1) in the place of *Hundreds*. Which done, the true Summe or Aggregat of all the Summes, appears in the ſeveral *Windows*, *viz*. 4646 *l*.—— 12 *s*.—— 3 *d*.—— 3 *f*. As in Fig. *C*. The ſame thing may be performed by adding together, Firſt the *Farthings* apart, then the *Pence*, next the *Shillings*, and laſtly the *Pounds*.

If peradventure the Summes are ſo numerous, as that by often turning of the Plates with holes, the ſmall Plates above them are like to be over-charged, That is, if the Figure (7) (8) or (9) of thoſe ſmall Plates come between the 2 *Lines* or *Marks*, then I diſcharge them, by turniug on the ſame numbers in the next places: For example, if the Figure (7) of the ſmall Plate above the place of *Farthings*, be under the *Line*, I turn on (7) in the place of *Pence*. If the Figure (7) of the ſmall Plate above the *Pence* be between the two *Lines*, I turn on (7) in the place of *Shillings*, and ſo to the end. And laſtly, I turn ○ of each ſmall Plate under its proper *Line*, And hen proceed (without ſetting the lower Plates to (0) or at all altering them otherwiſe than aforeſaid) to ſet on the remaining Summes, to a *Million* of Pounds. (Or if it be deſired, the Inſtrument may be made for a far greater Summe.) But the ſureſt way, is to divide a long Page into two or three parts, and ſo to work them diſtinctly.

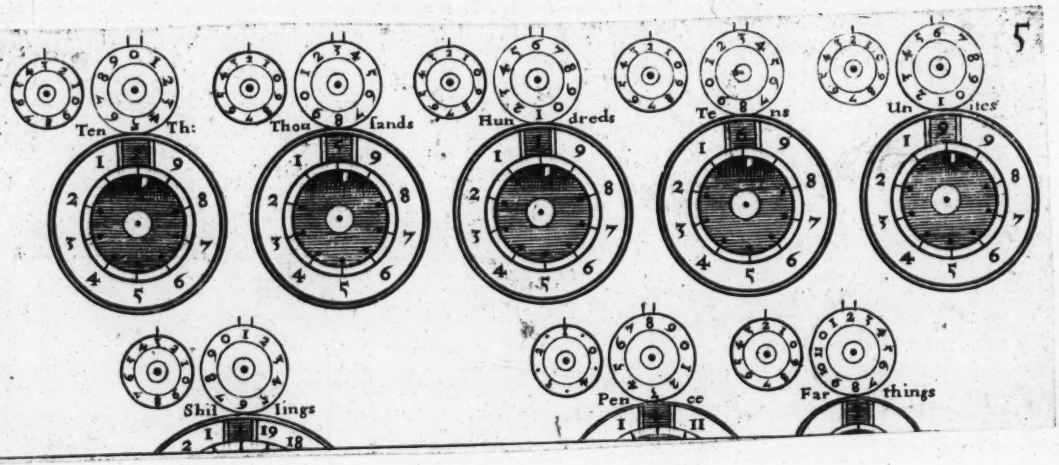

D
Ten Thouſ:
Thou ſands
Hun dreds
Te ns
Un ites
Shil lings
Pe nce
Far things
E
Ten Thouſ:
Thou ſands
Hun dreds
Te ns
Un ites
Shil lings
Pe nce
Far things

The Operation of Subſtraction.

Having prepared the Inſtrument for *Subſtraction* as is before directed, ſuppoſe I would from 327 *l.*-12 *s.*-07 *d.* deduct 39 *l.* - 14 *s.* - 3 *d.*

Firſt, I ſet on 327 *l.* - 12 *s.* 7 *d.* in their proper places. Then I begin with 3 *d.* of the Summe to be deducted, and ſet it backward, That is, I put the point of the *Index* into the hole under the *Window* of *Pence*, and turn it till it ſtand over againſt (3) in the Margent, which will leave (4) in that *Window*. Thus I ſet 14 *s.* backwards in the place of *Shillings*; And thus (39) in the *Unites* and *Tens* of Pounds. And having ſo done, I obſerve if any (0) of the ſmall Plates over head be removed from under its *Line* (as in Fig. *D.* in the place of *Shillings*, I find the ſmall Plate over head moved) I put the point of the *Index* into that hole of the next place, that is under the *Window*, and turn it backwards under (1) in the Margent. After the ſame manner, becauſe the ſmall Plate over the place of *Unites* of Pounds is removed out of its place, I ſet back for it (1) in the place of *Tens*; And ſo likewiſe, becauſe the ſmall Plate over the place of *Tens* is removed, I ſet back (1) in the place of *Hundreds*. Which done, the remaining Summe, *viz.* 287 *l.* - 18 *s.* - 04 *d.* is found in the reſpective *Windows*. As in Fig. *E.*

F.

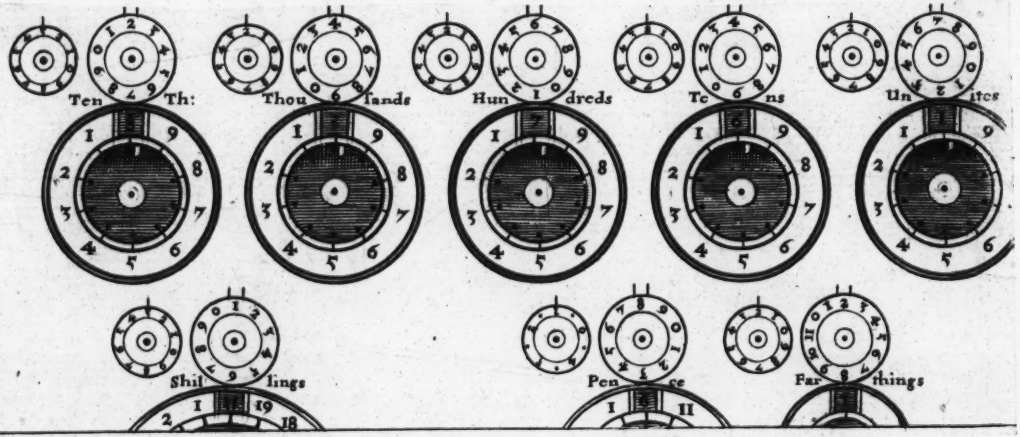

Ten Th:
Thou sands
Hun dreds
Te ns
Un ites
Shil lings
Pen ce
Far things

The Deſcription and Uſe of an *Additional Wheel*, applicable to this New *INSTRUMENT*, for all thoſe who ſhall deſire it, which renders it uſeful, beyond exception, for the longeſt Accompts, without either dividing the Page, or ſetting ⊙ of any of the ſmall Plates under their Lines a ſecond time.

BUt *foraſmuch as it may and will often happen, in the adding up long Pages of* Accompt-Books, *That the ſmall Plates will be over-charged (and that more than once in a* Page) *which may be ſomething troubleſome to the* Operator; *Therefore I have contrived an Additional* Wheel, *as in Fig.* F. *ſo that no one or two Pages of any* Accompt-Book *whatſoever can overcharge it: all the difference is only this, That what Figure ſoever of this* Wheel *is found oppoſite to the* Line (*or mark*) above it, *it muſt be ſet on* (*not in the next place, but*) *in the next place ſave one, in the manner following: That is to ſay, The Figure of the ſmall* Wheel *above the* Farthings, *oppoſite to the* Line above it, *muſt be turned off in the place of* Shillings. *The Figure of the ſmall* Wheel *above the* Pence, *oppoſite to the* Line above it, *muſt be turned off in the place of* Unites *of* Pounds. *And if any Figure of the ſmall* Wheel *be half a diviſion beyond the ſaid* Line, *there muſt be* 10 *turned off for it in the place of* Shillings, (*which is the greateſt intrigue in the whole Operation, For,*) *The Figure of the ſmall* Wheel *above the place of* Shillings *oppoſite to the* Line, *muſt be turned off in the place of* Tens *of* Pounds. *The Figure of the ſmall* Wheel *oppoſite to the* Line, *in the place of* Unites *of* Pounds, *muſt be turned off in the place of* Hundreds. *Laſtly, the Figure of the ſmall* Wheel *oppoſite to the* Line, *above the place of* Tens *of* Pounds, *muſt be turned off in the place of* Thouſands; *and ſo to the end, as in Fig.* G. *So that by the addition of this ſmall* Wheel, *a*

place

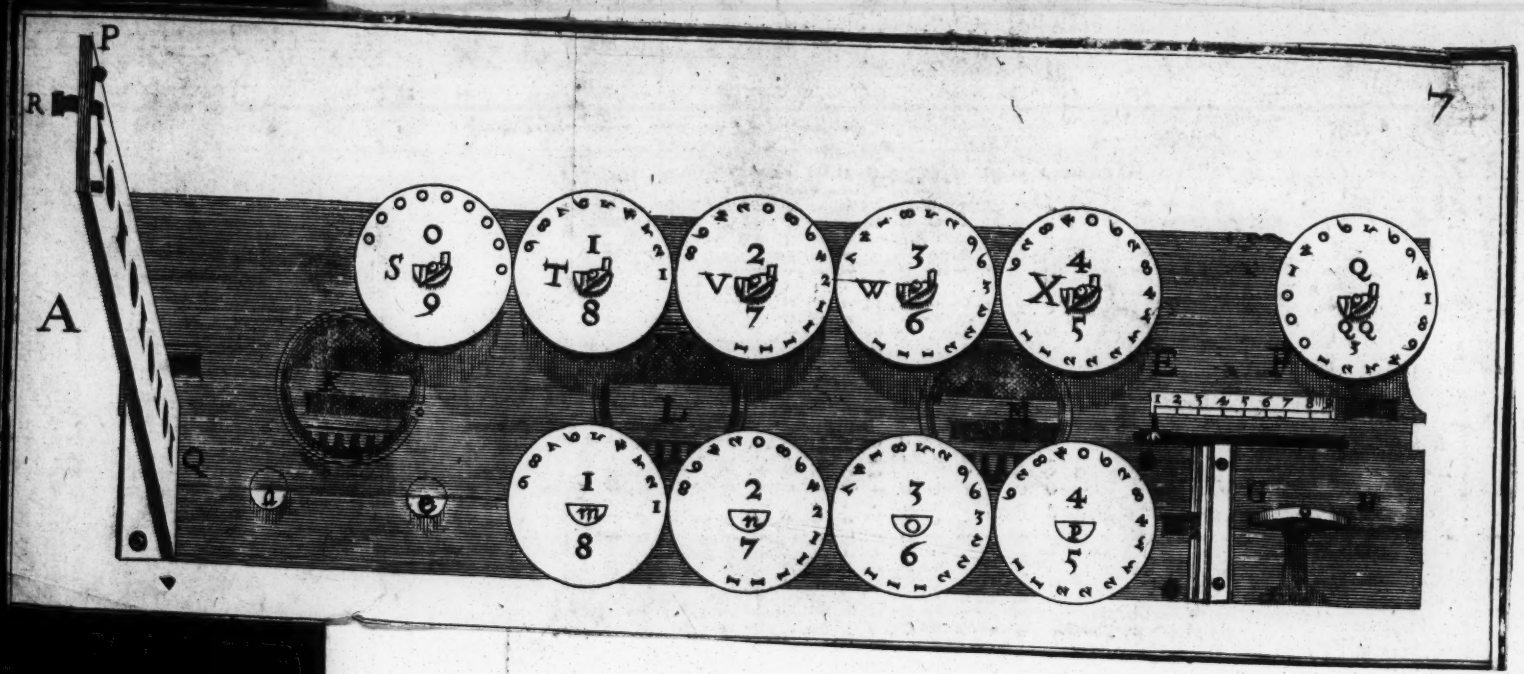
G


Ten Th:
Thou sands
Hun dreds
Te ns
Un ites
Shil lings
Pen ce
Far things

place *more is gained, and the* Inſtrument, *if need were, would add up an* Accompt *of* Ten Millions *of* Pounds, *by the* Example, *compared with* Fig. G. *will more evidently appear.*

EXAMPLE.

l.	s.	d.	q.
76534	13	03	3
76534	10	11	2
85637	14	05	1
93792	17	10	2
74379	09	08	3
85466	19	10	2
72954	10	04	2
61117	16	08	3
52252	16	06	3
65577	15	05	3
42573	15	02	3
72576	19	09	2
69955	17	11	3
72777	13	09	1
85855	04	10	3
98888	14	06	2
97744	16	07	2
45757	12	04	3
73879	16	11	3
64549	19	08	2
97872	14	06	1
85678	10	10	1
76644	08	06	3
56279	18	03	2
45772	15	05	3
56572	17	10	2
67699	14	11	2
71578	18	08	2
89979	19	10	3
97979	01	06	3
95878	08	11	2
86788	10	09	1
99678	13	10	2
89485	17	06	2
47632	16	03	1
57416	15	09	3
62517	14	07	2
74528	13	11	2
43215	12	05	2
67742	11	08	3
35418	09	04	1
92261	01	09	3
44415	12	06	3
43324	14	03	1
37338	02	11	2
34512	19	04	2
56735	17	02	3
3283761	11	06	1


7
P
R
A
Q
S
T
V
W
X
L
E
F
1 2 3 4 5 6 7 8

Machina Nova

CYCLOLOGICA

Pro Multiplicatione.

OR,

A new Multiplying-

INSTRUMENT:

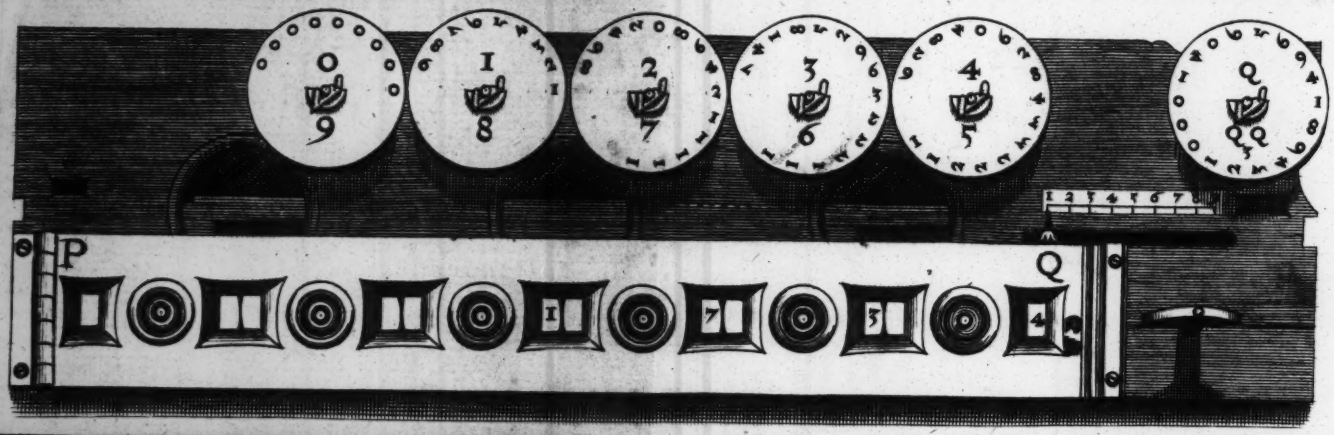
Invented, and humbly preſented to the Kings moſt Excellent MAJESTY

CHARLES II.

By S. MORLAND, *1666.*

THe Fabrick of this Inſtrument being truly repreſented in *Perſpective*, there will be no need of any large Deſcription of it.

The 5 upper Circles whereon are the Figures $\frac{0}{9}\frac{1}{8}\frac{2}{7}\frac{3}{6}\frac{4}{5}$ repreſent 5 moveable Plates which lie always ready to be taken off the 5 Semi-circular Pinions (or Centers) *S*, *T*, *V*, *W*, *X*, and to be ſet on any of, or all the Centers (or Semicircular Pinions) *a*, *e*, *m*, *n*, *o*, *p*, interchangeably. And underneath each of thoſe upper


B
P
Q

The Multiplying Inſtrument.

Plates aforeſaid,, are placed 5 other Plates of the ſame dimenſions, and with the ſame Figures graved on both ſides; ſo that of theſe upper Plates there are 30, that is to ſay, Six of each kind, beſides 5 other Plates under the place marked $\frac{9}{99}$ which ſerve for extracting the Square, Cube, and Square-Square-Roots, of which in its place.

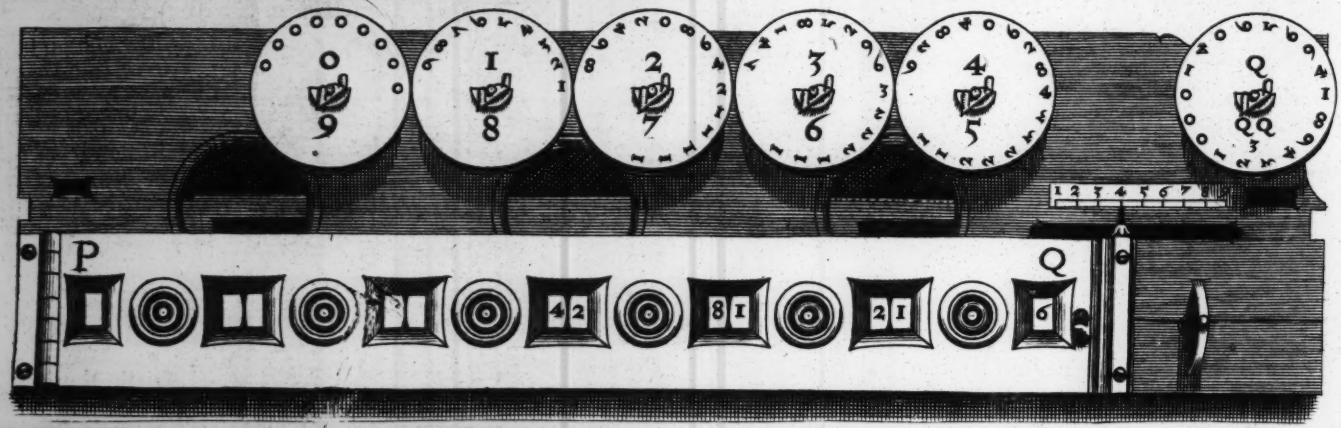
E. F. is a Line divided into 9 equal Parts, on which runs a ſmall Black Pin or *Index* forwards or backwards at pleaſure, being turned about by *G. H.* which is like to the Key of a Watch.

Laſtly, *P Q* is a Plate opening with hinges, and ſhutting down upon the lower Circular Plates, and a little Bolt at *R.* locking it down, which Plate has 6 ſquare holes (or *foramina*) through which are diſcovered ſuch Figures onely, as are neceſſary for the operation.

K, L, M, is a long Rack on the backſide of the Inſtrument, by which all the Plates are turned about on the Pinions a, e, m, n, e, The meaning of all which is this. The Inſtrument is to be ſuppoſed as it is repreſented in Figure *A* ſave onely that there are as yet no Circular Plates taken off the Semi-circular Pinions *S, T, V, W, X,* and that the number given to be multiplied is (1734.) Then one of the Plates marked $\frac{4}{4}$ is firſt to be taken off the Pinion x and ſet on the lower Semi-circular Pinion (r) which is the furtheſt on the right hand. Afterwards one of the Plates marked $\frac{3}{3}$ is to be taken off the Pinion *W*, and ſet on the Pinion (o) next to the other.

Thirdly, one of the Plates marked $\frac{7}{7}$ is to be taken off the Pinion *V* and ſet on the Pinion (n.)

Laſtly, one of the Plates marked $\frac{1}{1}$ is to be taken off the Pinion *T*, and ſet on the Pinion (m) Which done, the Plate *P Q.* is to be ſhut down, and then will appear the number 1734 through the holes, as in Fig. *B* and this is all the trouble of the Inſtrument in this or any operation.


C
P
Q
42
81
21
6

The Multiplying Inſtrument.

The Figure *B* repreſents the very ſame poſture of the Inſtrument in the foregoing Figure *A* ſave onely that the Plate *P Q* is now ſhut down and lockt upon the four lower circular Plates marked on the former Figure *A* with the large Figures $\frac{1}{8}$ $\frac{7}{2}$ $\frac{3}{6}$ $\frac{4}{5}$ which was in effect, a preparation for the Multiplication of the Number (1234) which Number now appears through the ſmall holes or *foramina* of the ſaid Plate *P Q* hiding all the other Figures from the Eye of the Operator. In which particular this *Inſtrument* far ſurpaſſes the Lord *Napiers Lamina* or *Bones*, which expoſe a great number of Figures to the Eye at the ſame inſtant of time (as well thoſe that are not uſeful as thoſe that are) beſides that all the Figures in thoſe Bones are placed Diagonally, which does very much ſtrain and force the Eye of the Operator; Whereas in this they lie all in a ſtraight Line, and as diſtinctly as can be deſired.

The Inſtrument being diſpoſed as aforeſaid, ſuppoſe the given Number 1734, be to be multiplied by 24. I firſt ſet it down in Writing thus

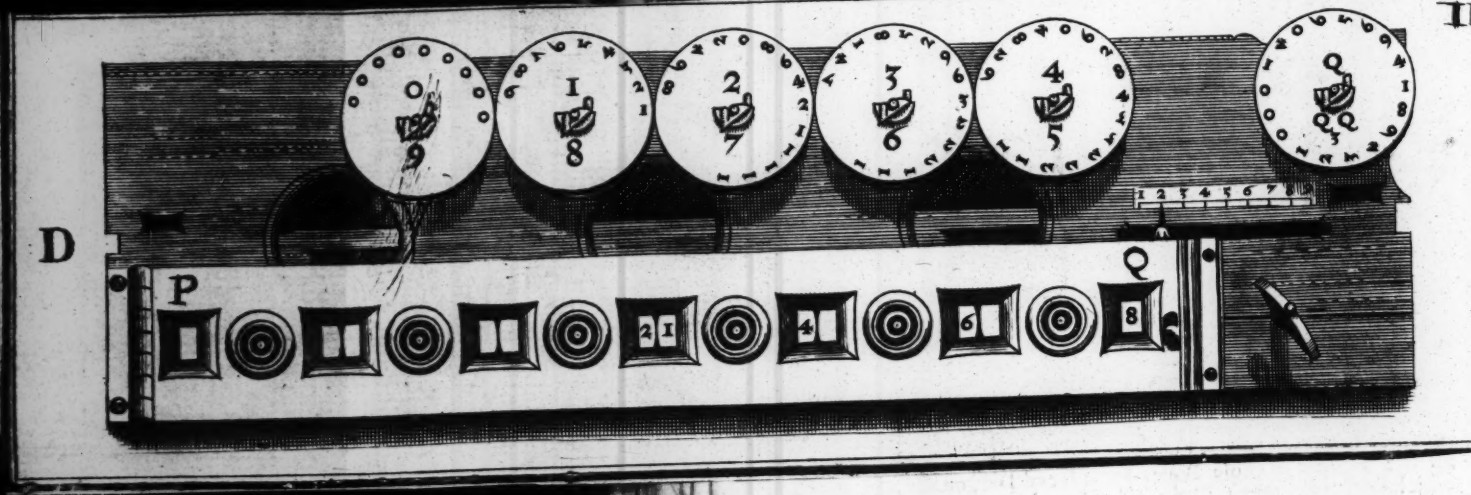
1734
24

then firſt I turn then Hand or Key *G H* till the *Index* point to 4 in the *Line E F* ſo have I the firſt Product (6936) given me in the Windows or *Foramina* of the Plate *P Q*, as in Fig. *C* which I ſet down thus

1734
24
6936

This done, I turn the Hand or Key backward, till the *Index* point to the Figure (2) in the Line *E F* and in the Holes or *Foramina* of the Plate *P Q* (in Figure *D*.)

3468
41616


D
P
Q

The Multiplying Inſtrument.

The Second Product, *viz.* 31468. which I ſet down under the firſt Product, and add all of them as in the ordinary Method of *Multiplication*, or elſe by the help of *Napiers* Bones.

1734	
24	
6936	
3468	
41616	

And after this manner may any Number be Multiplied by the help of this *Inſtrument*, which is capable of being made for any Number of places.

This *Inſtrument* alone is alſo of excellent uſe in *Diviſion*; as likewiſe in extracting the Square, Cube, and Square-Square-Roots, for which the 6 Plates at the end of the *Inſtrument* on the right hand are extreme ready and ſerviceable, and are to be uſed after the very ſame manner as are the Lord *Napiers* Bones, and therefore need no further explication.

But if any perſon have the Curioſity, and is willing to goe to the Expence; the *Adding Inſtrument* being joyned to the *Multiplying Inſtrument*, performs *Addition*, *Subtraction*, *Multiplication*, and *Diviſion*; as likewiſe the Extraction of the Square, and Cube-Roots, *&c.* without the help of Pen and Ink, or expoſing the Operator to any difficulty or uncertainty.

SOLI DEO GLORIA.

Ma-

A

CAUTION

To all who deſire to make Uſe of either of theſe

INSTRUMENTS.

IF any perſon deſire to have either of theſe Inſtruments *exactly made, and ſo as it may be ſerviceable for many years. He may beſpeak it of Mr.* Humphry Adamſon, *living at preſſent at the Houſe of* Jonas Moor, *Eſq; in the* Tower, *who is the onely Workman that ever as yet could be found by the Author to perform the ſaid* Inſtrument, *with that exactneſs that is abſolutely neceſſary for ſuch Operations.*

FINIS.

A Short DISCOURSE Concerning the ANTIENT, & COMMON WAY OF NUMBRING.

BUt for the better understanding of these Arithmetical *Instruments*, I shall endeavor so to explain and demonstrate the reason of the Operations of *Addition*, *Subtraction*, *Multiplication*, *Division*, and *Extraction* of the Square, and Cube-Roots, as to render them plain and obvious to the meanest capacities.

The way of *Numbring* in use with the Antients, was by the Letters of their respective Alphabets; For example: with the *Romans* C signified 100. D 500. M 1000. ↄMC 10 000. CC. I ↄↄ 20 000. C q ↄ 100 000. &c.

So among the *Grecians*, A or α. signified 1. B or β 2. T or . 3. I ι. 10. P ρ. 100. α, 1000. δ, 4000. ζ 7000. &c.

But note that the most common way of expressing the value of *Numbers*, is by the *Arabick* Notes or Characters called *Ziphers*, by the *Hebrews*, *Sephers*; and by Us, *Cyphers*; and they are ten, *viz.* 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

The way of placing those Characters, is from the right hand to the left, after the manner of the *Hebrews*, in their Writings, as *Gemma Frisius* and others observe.

The

The Progreſſion of them is *Decimal*, or by *Tens*; for every place to the left, is ten times the value of the next place to the right, as in the following Table.

The Third Period			The Second Period			The firſt Period		
Hund. of Millions	Tens of Millions	Unites of Millions	Hund. of Thouſ.	Tens of Thouſ.	Unites of Thouſ.	Hund. of Unites	Tens of Unites	Unites of Unites
c M	x M	i M	c M	x M	i m	c u	x u	i u
9	8	6	1	5	7	4	3	2
Nine H. Eighty ſix Milli.			One H. fifty ſeven Thouſ.			Four H. thirty two Unites.		

In this Table, the firſt Period is of *Unites*, the ſecond of *Thouſands*, the third of *Millions*, Thoſe who would proceed to more Periods then what are here ſet down, may give them what appellations they pleaſe, as *Billons*, *Trillions*, *Quadrillians*, *Quintillians*, &c. as *Tacquet* and others have done before us.

But the beſt and ſureſt way of placing and diſtinguiſhing great *Numbers*, is to put a *Comma*, or other mark of diſtinction betwixt every three of them, thus, 12, 345, 769.

CHAP.

CHAP. I.

The Precept for ADDITION *of Integers in Plain* Numbers.

HAving placed the Unites of the reſpective Progreſſions in Ranks and Files; then begin and add together the Unites of the right-hand-File, ſetting down the ſum underneath, if it be under 10. but if juſt ten, ſet down 0. and carry 1. to the next place; and if above 10. ſet down the exceſs in the firſt place, for every 10. an Unite.

Examples.

To 23	2374
add 14	1067
Summ 36	399
	3840

CHAP. II.

The Precept for SUBTRACTION *of Integers in Plain* Numbers.

HAving placed the leſs *number* under the greater, according to the reſpective places or Progreſſions, begin at the right hand, ſubſtracting the lower Figure out of that above it, and ſetting the Remainder underneath. But if the Figure chance to be the leſs, (as oft it falls out) then there muſt be an Unite borrowed, and brought from the next Progreſſion, to ſupply the defect, which muſt be again repaid, by adding an Unite to the next lower Figure on the left, which is the ſame thing

thing as if the Figure above it were diminiſhed by an Unite. And for a proof of the operation, the *number* Subtracted, and the *remainde*, muſt evermore equal that out of which it was Subtracted.

For out of a *number* A C let a leſs *number* A B be deducted, Then by the *Hypotheſis*, A B together with the *remainder* B C are equal to the whole *number* A C Forasmuch as *The parts united, are always equal to the whole.*

A I I I I I I I I B I I I I I I I I I I I I I I I C

Examples.

from 28	from 34	from 405
ſubtract 15	ſubtract 7	ſubtract 26
Remains 13	Remains 27	Remains 379

from 324531
ſubtract 99988
Remains 224543

CHAP III.

The Precept for MLTIPLICATION *of Integers in Plain* Numbers.

HAving placed the *Numbers* one under the other as in *Addition*, Multiply the laſt right-hand Figure of the *Multiplicand*, by the laſt right-hand Figure of the *Multiplicator*, and ſet the Product (if leſs then 10) underneath; but if greater, carry the exceſs (that is for every 10, an Unite) to the next place. And if the *Multiplicator* have more places then one, ſet down the firſt Figure of each reſpective Product under that Figure of the *Multiplicator* by which it was made, and ſo on to the left: obſerving *Ranks* and *Fyles*.

For

For example, 7 times 6 is 42, that is ſetting down 2 in the place of Unites and carrying 4 in the mind to the next place. Again 7 times 2 is 14, and 4 that was carried in mind makes 18, that is ſetting down 8, and carrying 1 to the next. Again 7 times 4 is 28, and 1 is 29.

And ſo in the ſecond Product twice 6 is 12, that is 2 carrying 1 to the next, and ſo on to the end.

426	Multiplicand.
327	Multiplicator.
2982	Firſt Product.
852	Second Product
1278	Third Product.
139302	Summe of all three Products.

The reaſon of which operation is plain by the following *Table*.

	CM	XM	M	C	X	U.	
				4	2	6	Multiplicand.
				3	2	7	Multiplicator.
For 7 times 6 is 42; and 7							
times 20 is 140; and 7					4	2	First Product
times 400 is 2800, or two				1	4	0	Second Prod.
thousand eight hundred.			2	8	0	0	Third Product
Again, 20 times 6 is 120;							
and 20 times 20 is 400;				1	2	0	Fourth Prod.
and 20 times 400 is 8000				4	0	0	Fifth Product
or Eight thousand.			8	0	0	0	Sixth Product
Thirdly, 300 times 6 is							
1800; 300 ttimes 20 is			1	8	0	0	Seventh Prod.
6000; & 300 times 400 is			6	0	0	0	Eighth Prod.
120000, or One hundred	1	2	0	0	0	0	Ninth Prod.
and twenty thousand.							
The sum total of which							
Products amounts to							Summe total
One Hun. thirty nine Th.	1	3	9	3	3	2	of all nine
three Hund. and thirty							Products.
two Unites.							

CHA

CHAP. IV.

The Precept for DIVISION *in Plain* Numbers.

DIVISION, is in effect nothing else, but the deducting of a less *number* as oft as may be out of a Greater, and so finding at last the *number*, by whose Unites that less number being repeated, makes a *number* equal to the Greater.

Now the greater of these *numbers* is Vulgarly called the *Dividend*, the less the *Divisor*, and the last the *Quotus* or *Quotient*.

The method of this Operation is thus;

1. Set the Figures of the *Divisor* under an equal *number* of Figures of the *Dividend* on the left hand, if those Figures of the *Dividend* be of greater, or at least of equal value with those of the *Divisor*: Otherwise you must place the first Figure of the *Divisor* under the second Figure of the *Dividend*. And having set the *Divisor* right, put pricks over the Figures of the *Dividend*, from the Unite place of the *Divisor*, inclusive. And the *number* of pricks denote the *number* of places in the *Quotient*.

2. You must evermore prepare such a *Tariffa* (or Table of *Multiplication*) for the *Divisor*, as is here set down on one side of the *Operation*, and is of excellent use, making the work ten times more easie and certain.

3. You must find by the *Tariffa* how many times the *Divisor* is found in those Figures of the *Dividend* under which they are placed, and the answer to that, is the first Figure of the *Quotient*; by which you have multiplied the *Divisor* in the *Tariffa*, then deduct the product out of those upper Figures of the *Dividend*, and what remains must be considered in the next *operation*, if there be more places then one in the *Divisor*.

4. The next Figure of the *Dividend* must be taken down and set next to the Remainder, if there be any. And the *Divisor* must be again set under it, if the value

of

of the upper Figures be ſufficient; if not, there muſt a Nullor (0) be ſet in the *Quotient*, and then the next Figure of the *Dividend* taken down, and the very ſame Operation repeated, till the work be at an end. But one Example in things of this nature clearly and diſtinctly ſet down, is better then a thouſand verbal directions.

Let the *Dividend* be that *Number*, which was laſt found by Multiplying (426) by (327)

That is to ſay,

Let the *Dividend* be 139302 (426
And the *Diviſor* be 327
1308

Having pointed the *Dividend*, and placed the *Diviſor* under (1393;) look for 1393 (or the neareſt number to it) in the *Tariffa*, which is 1308. wherefore I ſet that down; and ſubtracting it from 1393, there remains 85; then (having ſet down 4 for the firſt Figure of the *Quotient*) I take down the next Figure, or Cypher of the *Dividend*, viz. (0) which makes it (850) In this (327) the *Diviſor* by the aforeſaid method is found twice; wherefore I ſet (2) in the *Quotient*, and then deduct the Product, viz. (654) out of it, and there remains (196) to which in the laſt place, I take down (2) the laſt Figure of the *Dividend*, and make it (1962) in which (327) is found 6 times, and ſo the work is at end.

850
327
654
1962
327
1962
0000

Tariffa.

1	327
2	654
3	981
4	1308
5	1635
6	1962
7	2289
8	2616
9	2943

The

The reason of this Operation is plain in the subsequent Table.

C X
M M M C X U

	1	3	9	3	0	2	*Dividend.*
		3	2	7			*Divisor*, which is advanced 3 places, and is not now 327. but 32700.
Quot. 400.	1	3	0	8			This *Product* is not 1308, but 130800, which is 4 times 32700, that is 420 times 327, which is the *Quotient* in the Margent.
			8	5			That is 32700 being deducted out of 130800, the remainder is 8500, and so is ended the first Operation.
			8	5	0		Here begins a second Operation. And this is not 850, but 8500.
				3	2	7	This is not 327 but 3270, which being doubled makes 6540, which in effect is 20 times 327, as is expreſt by the *Quotient* in the Margent.
Quot. 20.				6	5	4	
				1	9	6	That is, 6540, being deducted out of 8500, there remains 1960, and so ends the second Operation.
				1	9	6	2	
					3	2	7	Here 327 is brought down to its own value again.
Quot. 6.				1	9	6	2	That is 6 times 327 is just equal to 1962, and so the work is at end.
				0	0	0	0	

But now if this *Dividend* had been greater by 20 Unites, that is, if it had been (139322) the work had been the ſame, and the *Quotient* had been the ſame *number* of Integers, but there had been found remaining a broken part or Fraction of 20. which muſt have been be ſet thus $\frac{20}{327}$

CHAP V.

Notation of FRACTIONS.

A Broken *Number*, (otherwiſe called a *Fraction*) is part of an Integer; For example, A Foot in length contains 12 Inches. One pound 20 ſhillings, One ſhilling, 12 pence; One penny, 4 Farthings, *&c.*

The Parts of a Fraction.

A *Fraction* conſiſts of 2 parts, The *Numerator* and the *Denominator*, which are placed one above the other, and ſeparated by a little line; For example, If I would expreſs three quarters of a yard, it muſt be ſet thus

$\frac{3}{4}$ *Numerator*
Denominator.

F. Inch.

Three Foot and nine Inches, thus. 3. $\frac{9}{12}$ *Numerator.* *Denominatorr.*

Or if a Foot be divided into an hundred parts, thus $3\ \frac{75}{100}$ *Num.* *Denom.*

Or if a Foot be divided onely into four parts, thus $3\ \frac{3}{4}$ *Num.* *Denom.*

And ſo are all other broken Parts or *Fractions* expreſt of what kind ſoever.

CHAP.

CHAP. VI.

The Reaſon of Tranſlating Fractions *from one Denomination to another, as likewiſe of reducing them to their leaſt Tearms, and the truth of the Operations demonſtrated from ſeveral Propoſitions of* Euclids *Elements.*

HAving thus explained the nature of *Fractions*, I ſhall in the next place proceed to demonſtrate the Reaſon of Tranſlating *Fractions* from one *Denomination* to another, as likewiſe the Reaſon of depreſſing or reducing them to their loweſt and leaſt Terms. All which is neceſſary to be known by thoſe who deſire to be Maſters of *Fractions*.

Firſt of Tranſlating *Fractions* from one *Denomination* to another.

Eucl. l. 7. Theor. 16. Prop. 18.

Ἐὰν δύο, &c.

If two *numbers* multiplying any *number*, produce other *numbers*, the *numbers* produced of them, ſhall be in the ſame proportion that the *numbers* multiplying are, *vid. Fig. E.*

For the applying of this Propoſition to the matter in hand, I lay before me theſe two *Fractions*, namely $\frac{2}{3}$ and $\frac{3}{4}$ as *in Fig. F.* And firſt I ſingle out $\frac{2}{3}$ and Multiply them by (4) the *Denominator* of the next *Fraction* and they become $\frac{8}{12}$ for 4 times 2 is (8) and 4 times 3 is (12) as may be ſeen in *Fig. G.*

E

Multiplicans = 3

Multiplicandus = 4

Ut Multiplicandus = 4
ita Productus = 12

E

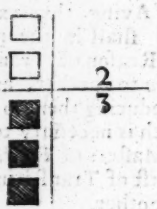
Multiplicandus = 5.

Multiplicans

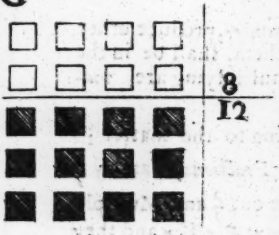
ad Multiplicandum = 5
ad Productum = 15

F

F



G



H

This done, I ſay, that by the afore-mentioned *Propoſition*, this Multiplication of the two *numbers* (2) and (3) by a 3d *number* (4) has altered no proportion, but the products have the ſame proportion one to another, as the *numbers* (2) and (3) have, by which they are multiplied, that is to ſay, as (2) is to (3) ſo is (8) to (12).

Again, I ſingle out the other *Fraction*, *viz*: $\frac{3}{4}$ and I multiply theſe two by a third *Number*, namely by (3) the firſt *Denominator*. And the Products are $\frac{9}{12}$ as in *Fig. H.* Neither hath this Multiplication altered any proportion, by the ſame reaſon with the former. For as (3) is to (4) ſo is (9) to (12). By which means I have two new *Fractions*, *viz*. $\frac{8}{12}$ and $\frac{9}{12}$ which are the ſame in effect with the firſt two, *viz*. $\frac{2}{3}$ and $\frac{3}{4}$. And whereas they were before of different *Denominations*, they are now brought under one and the ſame *Denomination*.

And thus is that 18*th Prop.* of the 7*th* Book of *Euclid*, the true ground and reaſon of altering and tranſlating *Fractions* from one *Denomination* to another; And from divers and different *Denominations*, to one and the ſame. Which *Propoſition* being once throughly digeſted, and imprinted in the mind and memory, all other Operations which relate to *Fractions*, (as Adding, Subſtracting, Multiplying, and Dividing them, as alſo extracting their *Square* and *Cube Roots*) will admit of very few or no difficulties.

Secondly, *The way of reducing* Fractions *to their leaſt Terms.*

But foraſmuch as it often happens in the multiplying and tranſlating of *Fractions* that they ſwell into too great *numbers*, which are not ſo tractable as ſmaller *numbers* are. It will be proper in the next place, to ſhew the way of reducing them to their *leaſt terms*, either before, or after they are thus multiplied or tranſlated, as the practitioner ſhall ſee occaſion. For the better effecting of which, he is deſired to conſider ſome few *Definitions* and *Problems* of the 7*th* Book of *Euclid's Elements*,

Eucl. l. 7.

Defin. XI.

Πρῶτος ἀριθμὸς, &c.

A prime *number* is that which is meaſured onely by an Unite.

That is to ſay 2, 5, 7, 11, 13, &c are *prime numbers*, becauſe neither of them can poſſibly be divided into equal parts by any thing leſs then an Unite.

Defin. XII.

Πρῶτοι πρὸς, &c.

Numbers prime the one to the other, are ſuch as only an Unity doth meaſure, being their common meaſure.

And ſuch are (9) and (14) for theſe two *numbers* cannot be divided into leſs then Unite parts, ſo as that which meaſures one, may meaſure the other.

For though (3) will *meaſure* ⟨ 111111111. ——— 9 9, that is to ſay, it is found 3 ⟩ 11111111111111. 14 times in (9) yet it will not *meaſure* (14) that is, after it has been found as many times as it can be found in (14) there will be two odd Unites left.

Again though (2) do *meaſure* (14), that is to ſay, is found juſt 7 times in (14) yet it cannot be found any certain *number* of times in (9) but there will be an odd Unite or Unites left.

Defin. XIII.

Σύνθετος ἀριθμὸς, &c.

A compoſed *number* is that which ſome certain *number* meaſureth.

So is (15) a *compoſed number*, becauſe (3) multiplyed by (5) makes (15). And ſo is (20) a *compoſed number*; becauſe 4 multiplyed by (5) makes a Product of (20).

Defin.

Defin. XIV.

Συνθετοι δὲ, &c.

Numbers compoſed, one to the other, are they, which ſome *number*, being a common meaſure to them both, doth meaſure.

And thus are (8) and (12) *compoſed Numbers* one to another, becauſe there is a certain *number*, *viz.* (4) which being repeated, or added to it ſelf a certain *number* of times, *compoſes* both the one and the other of theſe *numbers*. That is to ſay, (4) being repeated (or added to it ſelf) twice, *compoſes* (8) the one of theſe *numbers*, and the ſame (4) being repeated (or added to it ſelf) 3 times, *compoſes* (12) the other of theſe *numbers*. And this (4) is called the *common meaſure* of (8) and 12. as in the following Table.

Two compoſed *Numbers*	8 and 12
Diviſors of theſe 2 compoſed *Numbers*.	2 3
Their *common meaſure*.	4
Three compoſed *Numbers*	6 & 8 & 12
Diviſors of theſe 3 compoſed *Numbers*	3 4 6
Their *common meaſure*	2

Probl. I. *Prop.* 2.

Δύο ἀριθμῶν, &c.

Two *Numbers* being given (not prime the one to the other) to find out their greateſt common Meaſure.

For example, let the *Numbers* given be (A = 15) and (B = 9)

I take the leſs *Number* (B = 6) from the greateſt (A = 15) as oft as I can, which is once: If nothing remain then (B = 9) is the greateſt *common meaſure*. If ſomething? Let it be (C = 6), and then I

15 A IIIIIIIIIIIIIII
9 B IIIIIIIII
6 C IIIIII
3 D III
6 E IIIIII

take (C—6) out of (B—9) as oft as I can, and if there remains nothing, then (C—6) is the greateſt *common meaſure*; and if there remain ſomething, let it be (D—3) then I take (D—3) twice out of (C—6) and becauſe nothing remains, therefore (D—3) meaſures (C—6) and therefore it meaſures (B—9) and alſo (A—15) And is likewiſe the greateſt *Number* that meaſures B and A, that is to ſay their greateſt *common meaſure*. And this will be in all other *Numbers* (not *prime* one to another) before I can come to an Unity.

Probl. III. *Prop.* 35.

Αριθμῶν δοθέντων, &c,

How many Numbers *ſoever being given, to find the leaſt* Numbers, *that have the ſame proportion with them.*

Let the *Numbers* propounded be (A—6) (B—8) (C—12) either theſe are *Prime Numbers*, or elſe there is a certain *Number* which being repeated, (or added to it ſelf) a certain *Number* of times, compoſes either of them, and is therefore their *common meaſure*.

If they be *Prime Numbers*, I have granted what I deſire.

If not, let then their greateſt *common meaſure* be D. And look how many times D is found in A, let E have ſo many Unites. And let F have as many Unites as D is found times in B. And G as many Unites as D is found times in C. That is, (D—2) multiplied by (E—3) produces (A—6). And (D—2) multiplied by (F—4) produces (B—8). And (D—2) multiplied by (G—6) produces (C—12).

6 A IIIIII
8 B IIIIIIII
12 C IIIIIIIIIIII
2 D II
3 E III
4 F IIII
6 G IIIIII

Then E, F, and G, are the leaſt *Numbers* that have the ſame proportion with the firſt *Numbers*, A, B, and C.

CHAP.

CHAP. VII.

Addition of FRACTIONS.

LEt the given *Fractions* to be added together be $\frac{2}{3}$ and $\frac{3}{4}$.

Firſt I reduce them by the *6 Chapter*, to $\frac{8}{12}$ and $\frac{9}{12}$

Then I add 8 to 9, which makes it $\frac{17}{12}$ or $1\frac{5}{12}$.

If they be more then two. Firſt rednce two of them to one *Denomination*. And then the ſum of thoſe two, and the next, *&c.*

For example, Let the *Fractions* be $\frac{2}{3}.\frac{3}{4}.\frac{4}{5}$. Firſt, $\frac{2}{3}$ and $\frac{3}{4}$ make $\frac{17}{12}$. Then $\frac{17}{12}$ and $\frac{4}{5}$ being added as before, make $\frac{360}{60}$ and $\frac{85}{60}$. which make $\frac{121}{60}$ or $2\frac{1}{60}$.

CHAP. VIII.

Subtraction of FRACTIONS.

FRom $\frac{3}{4}$ are to be deducted $\frac{2}{3}$. That is by the foregoing Rules from $\frac{9}{12}$ are to be deducted $\frac{8}{12}$. Therefore there remains $\frac{1}{12}$.

CHAP. IX.

Multiplication of FRACTIONS.

THis is onely to *Multiply* the *Numerators* one by another, for a *New Numerator*, and then the *Denominators* one by another for a *New Denominator*.

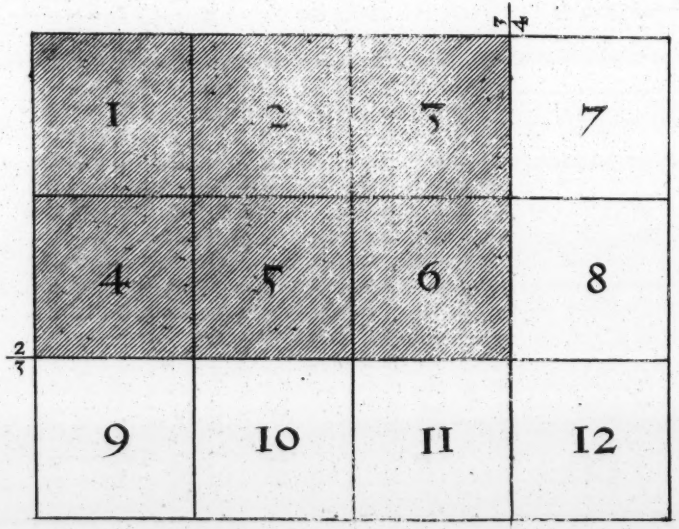
For

For example. To multiply $\frac{2}{3}$ by $\frac{3}{4}$ is to multiply 2 by 3. (6) for a new *Numerator*, and then 3 by 4 (12) for a new *Denominator*. This is $\frac{2}{3}$ by $\frac{3}{4}$ give $\frac{6}{12}$ or $\frac{1}{2}$ for the Product.

But to make all more plain, and to preſent both the Operation and the Reaſon of it, to the Readers eye, at one and the ſame inſtant; let him conſider the leſs *Oblong* (6) in *Fig. L.* (made by multiplying the *Numerator* (2) by the *Numerator* (3) included in the greater *Oblong* (12) made by multiplying the *Denominator* (3) by the *Denominator* (4) then which nothing can poſſibly be more plain, or ſatisfactory.

CHAP.

L



CHAP. X.

Diviſion of FRACTIONS.

THe Rule for *Diviſion* of *Fractions*, is this,

Multiply the Denominator *of the* Diviſor, *by the* Numerator *of the* Dividend, *for a new* Numerator; And *the* Numerator *of the* Diviſor, *by the* Denominator *of the* Dividend, *for a new* Denominator. *And the new* Fraction *is the* Quotient.

For example, if $\frac{3}{4}$ be to be divided by $\frac{2}{3}$

Then is $\frac{3}{4}$ the *Dividend*.

And $\frac{2}{3}$ the *Diviſor*.

Then 3 into 3 make a new *Numerator*, *viz.* 9
And 2 into 4 make a new *Denominator*, *viz.* 8 } that is $\frac{9}{8}$

That is to ſay $\frac{2}{3}$ is contained in $\frac{3}{4}$ once, and one eighth part. $1\frac{1}{8}$

The Reaſon of this operation is this.

Firſt, I turn the two *Fractions* $\frac{3}{4}$ and $\frac{2}{3}$ into two new *Fractions*, *viz.* $\frac{9}{12}$ $\frac{8}{12}$ (as before has been ſhown). Then, as if they were plain *Numbers*, I divide (9) by (8) and the *Quotient* is $(1\frac{1}{8})$. That is to ſay, the *Dividend* $(\frac{3}{4})$ is to the *Diviſor* $(\frac{2}{3})$, as $(\frac{9}{12})$ to $(\frac{8}{12})$. But now (9) is the Product of the *Diviſors Denominator* Multiplied by the *Dividends Numerator*, and is the *Numerator* of the *Quotient*, And (8) is the Product of the *Diviſors Numerator* Multiplyed by the *Dividends Denominator*, and is the *Denominator* of the *Quotient*; which is conſonant to the foregoing *Rule*, and that which was to be demonſtrated.

Thus

Thus if $\frac{6}{12}$ be to be divided by $\frac{3}{4}$.
$\frac{3}{4}$ into $\frac{6}{12}$ make a new *Numerator* 24 And $\frac{3}{4}$ into $\frac{12}{12}$ make a new *Denominator* 36 } *viz.* $\frac{24}{36}$
Which being reduced to its leaſt terms, is $\frac{2}{3}$

CHAP XI.

Of Decimal FRACTIONS.

THeſe Decimal *Fractions* are of all *Fractions* the moſt natural: For the truth is, all plain *Numbers* as they are expreſt by the *Arabick* Notes of 1, 2, 3, *&c.* are nothing elſe in effect, but Decimal *Fractions*. For example, Let any *Number* be given (432) The laſt Figure (2) is really $\frac{2}{10}$, and ſo are the laſt two Figures (32), truly and properly $\frac{32}{100}$. And all three are a *Fraction* of $\frac{432}{1000}$. For as the very progreſſion of theſe and all other plain *Numbers* is Decimal; that is, each Figure on the left hand, is ten times the value of the ſame *number* placed in the next place on the Right. Thus in (333) the laſt (3) on the left is ten times the value of the (3) next to it on the right; And the (3) in the middle, is ten times the laſt (3) on the right, and but the tenth part of that (3) on the left. And therefore all, or any of them may be *pro libitu*, either *Fractions* or *Integers*: If I would have them *Integers*, I ſet them down without any Line drawn under them; But if I would have the two laſt a *Fraction*, I put a ſeparating *Comma* between them, thus, 3, 33, that is 3 *Integers*, and $\frac{33}{100}$ for the *Denominator* is here to be underſtood to be an Unite of the next place or *Denomination*, as was before explained. And thus 5270 is an *Integer* or whole *number*, and the ſame *number* with a ſeparating

parating *Comma*, thus 5,270 is an *Integer* (5) with a *Fraction* of $\frac{270}{1000}$. Or the whole is a *Fraction* of $\frac{5270}{1000}$.

CHAP. XII.

Of Addition and Subſtraction of Decimals.

THe Operations of *Addition* and *Subſtraction* in *Decimals*, *Integers*, and *Fractions*, is the very ſame with that of plain *Integers*, only the careful ſetting the Unites of all the *Integers* in one File, and if there be any void places, they are to be imagined to be filled up with Cyphers.

Examples of Addition.

26,43	35,25	22,5
30,14	240,56	,1875
52,86	37,032	,0156
109,43	312,842	22,7031

Examples of Subtraction.

132,43	57,27	58,00	37,
24,26	38,43	27,92	0,104
108,17	18,84	30,08	36,896

CHAP. XIII.

Multiplication of Decimals.

THis is likewiſe the very ſame Operation with that of Multiplying plain *Integers*, ſave onely, when all

all the work is ended, there muſt be as many places of Parts, or *Fractions* in the Product, as there were places both in the *number* multiplied, and in the *number* multiplying.. As in the following Example, there are two places cut off in the *number* multiplied, and one in the *number* multiplying; Therefore there are three places cut off in the Product. And in the ſecond Example, becauſe there are three places cut off in the *Multiplicand*, and two in the *Multiplicator*: therefore there are five cut off in the Product.

1. *Example.*

32,12
24,3
9636
12848
6424
780,516

2 *Example.*

25,412
2,32
50824
76236
50824
58,95584

3 *Example.*

1,305 2 6
59,3
39157 8
783156
65,2635
7348613 8

4 *Example.*

,87
,9
,783

5 *Example.*

,0375
,05
1775
0000

5 *Example.*

,0375
,05
1875
0000
,001875

In the 4 Example, becauſe there are no *Integers*, the Product is $\frac{783}{1000}$

In the 5 Example, becauſe there do not ariſe but five places in the Product, *viz.* ,01875 and yet by the Rule there ought to be ſix places cut off, therefore two Cyphers

phers must be prefixt to make up the *number* of six places, *viz.* $\frac{001875}{1000000}$

CHAP. XIV.

Division of Decimal FRACTIONS.

1. IF the *Dividend* be greater then the *Divisor*, the *Quotient* will be either a whole *number*, or a mixt; but when the *Dividend* is less then the *Divisor*, the *Quotient* must be a *Fraction*.

2. Whatever the *Dividend* be, if need require, there must be a competent *number* of *Cyphers* added to it, so make room for the *Divisor* to stand under it, and then it is no other but ordinary *Division*, as will appear by the following Example.

Dividend 172, 5
Divisor 3, 746

which the *Dividend* being supplied with Cyphers, stands thus

```
      . . . . .
172,500000 (46049, &c.
 3746
14984
 22660
 22476
   18400
    3746
   14984
    34160
     33714 &c.
       446
```

Tariffa.

1	3746
2	7492
3	11238
4	14984
5	18730
6	22476
7	26222
8	29968
9	33714

CHAP.

CHAP. XV.

The Vulgar Precept for extracting the Square Root or ſide of any Plain Number.

HAving firſt pointed the given *Number*, (ſuppoſe 625) that is to ſay, ſett a prick or point over every other Figure, beginning with the laſt. Whereby I know that the Root has two places and no more. Firſt, I enquire whether (6) the laſt pointed Figure or Figures, be a true ſquare Number. If it be, then I ſet down the Root (which is eaſily found by the following Table, and indeed ought to be retained in the memory) ſome where on the right hand of the work, and ſo is the firſt Figure of the Squar-Root found. But if it be not a true Square *Number*, then I take the Root of the Square next to it, as in this caſe, I ſee (6) is not a true Square, I therefore take the Root of (4) which is the neareſt to it, and make that the firſt Figure of the Root. And then Subtracting (4 the Square thereof) out of (6) I have left remaining (2) which I ſet underneath, and ſo is the firſt Operation ended.

625 (2
4
2

This done, I take down the Figures of the next Square *Number*, *viz.* (25.) Then I ſet the double of the firſt Figure of the Root under the firſt Figure of the ſecond Square *Number*; and I ſeek out a Digit, which being multiplied into it ſelf, together with the double of the *Quotient* (or firſt Figure) may take away the remaining Figures of the given *Number*, *viz.* (225) or at leaſt as much as may be; which Digit, by asking how many times (4) the double of the Root is found in the Figures ſtanding over it, as in the nature of a *Dividend* (namely 22) for I find that (4) will be found in (22) 5 times, and enough over and above to

multiply that 5 into it self also. Therefore I set down (5) for the second Figure of the Root, and I also set the same (5) under the last Figure of the second Square; and then I multiply the double of the first Figure, augmented by that second Figure (5) that is, I multiply (45) by that (5) and if the Product be either equal to the Ftgures (225) standing above them, or so much less then (225) as the value of any *Number* under (45) then the second Figure is rightly chosen otherwise the Figure next less must be taken. But in this example, the Product happens to be just equal And so the work is ended.

```
 ·  ·  ·  ·
 6 2 5 ( 2 5
 4
 ---
 2 2 5
   4 5
 2 2 5
 -----
 0 0 0
```

Now in case this (625) had been (655) which is a greater *Number* then (625) by 30 Unites; the Integers of the Square Root had still been (25;) onely there had been left (30) which had been the *Numerator* of a *Fraction*, whose *Denominator* must evermore be the double of the Root, augmented by an Unite, and then the Operation had been, as here you see.

```
 ·   ·  ·  ·
 6 5 5 ( 2 5 30/51
 4
 2 5 5
   4 5
 2 2 5
 -----
   3 0
```

CHAP. XVI.

The Reason and Demonstration of the Vulgar Operation of Extracting the Square-Root.

AND after this very manner and method may the Square-Root of any Plain *Number* in Integers be extracted

extracted, though never ſo great; but that this and all other Operations of the ſame nature may be alſo performed with underſtanding, and ſatisfaction, it will be neceſſary to make ſome reflections upon the nature and geneſis of a Square-*Number*, and in order thereunto, the Practitioner is deſired to conſider the following Definition of a Square-*Number*.

Eucl. l. 7.

Defin. XVIII.

Τετράγωνος ἀριθμὸς ὅτάν, &c.

A Square-Number *is that which is equally equal, or, which is contained under two equal* Numbers.

THus the Square-*Number* (4) is contained under two equal *Numbers*, *viz.* (2) and (2) and the Square-*Number* (9) is contained under two equal *Numbers*, namely (3) and (3) and ſo on as in the following Table.

A Table of Squares with their Genetive equal *Number*.

Equal Number			Square
1	into	1	1
2	into	2	4
3	into	3	9
4	into	4	16
5	into	5	25
6	into	6	35
7	into	7	49
8	into	8	64
9	into	9	81
10	into	10	100

Thus the Square-*Number* (625) is contained under two equal *Numbers*, *viz.* (25) and (25) That is to ſay,

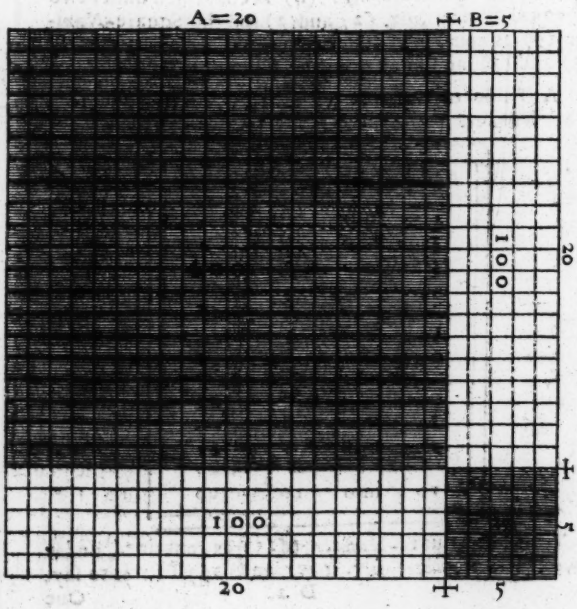
Sectio

QUADRATI (625)

in quatuor Plana, à duobus Lateris (25) Segmentis, viz. (A=20) & (B=5) effecta; quorum tria ordinatim ſumpta, ſunt continuè proportionalia,

nimirum

1 A quadratus = 400 = *maximus Proportionalis*
2 A in B = 100 = *medius Proportionalis*
3 B quadratus = 25 = *minimus Proportionalis*



One of the equal *Numbers* —— 25
being multiplyed by the other equal *Number.* —— 25

Makes the Product a Square *Number, viz.* —— 625

Euel. l. 2. Theor. 4. Prop. 4.

Ἐὰν εὐθεῖα, &c.

If a right Line be cut any wiſe into two parts, the Square made of the whole Line, is equal both to the Squares made of the Segments, and to twice a Rectangle made of the Parts.

THis holds good likewiſe in *Numbers.* For example, Let (25) be the *Number* of a Right Line, and that divided into two parts: *viz.* (20) the greater, and (5) the leſſer. I ſay,

The Square of the whole (25) is equal to the two Squares of (20) and (5) more by the two Oblongs made of (20) multiplied by (5) as in the oppoſite Figure may be plainly ſeen.

For let (20) be called —— A
And (5) be called —— B

A q: (or 20 multiplyed by 20) makes —— 400
B q: (or 5 multiplied by 5) makes —— 25
A, Multiplied by B, that is (20) multiplied by (5) } makes —— 100
A, Multiplied by B, that is (20) multiplied by (5) } makes —— 100

That is to ſay, theſe are the parts, which being united are equal to the whole Square } —— 625

Theſe things being premiſed, I ſay, that whereas the vulgar Rule directs (after the pointing of the *Number* 625 whoſe Square Root is to be extracted) to find out the Square-Root of the laſt pointed Figure on the left hand, that is (6) and not finding (6) a true Square *Number,* to take the next to it, *viz.* (4) whoſe Square-Root is (2,) I ſay, the meaning is this; That

(6) is in effect (600) which is not a true Square-*Number*, and the nearest to it is (400) whose Square-Root is (20.)

Again, whereas the vulgar Rule directs to subtract the Square-*Number* (4) out of (6) and so to find out how many times the double of the first Root is contained in the Remainder and the first Figure of the next Square, *viz.* (22) that is 5 times, But with this provision that there may remain a *Number* equal to the Square of that (5) as in the example.

The given Square-*Number*	6	25	The Binomial (A 2 | B 5) Root
(A = 20) Squared	4	00	
Remainder	2	25	
Double of the first Figure		40	
(A = 20) into (B = 5) twice	2	00	
(B = 5) Square^{d}		25	
Aggregate	2	25	
	0	00	

The meaning is this, Having Subtracted A q, or the Square-*Number* (400) out of the Number (625) there remains (225) which is (A = 20) into (B = 5) or (100) for one of the Oblongs, and (A = 20) into (B = 5) for the other Oblong, and B q or (25) for the lesser Square. All which are the very parts of a Square, expressed in the foregoing Proposition of *Euclid*. Namely.

(A q) The Square of the greater Segment which is equal to	400
(B q) The Square of the lesser Segment which is equal to	25
(A) Into (B) one of the Rectangles made of the Segments, and equal to	100
(A) Into (B) the other Rectangle made of the Segments and equal to	100

The

The Square made of the whole (A-|-B) or (25) multiplied by (25) is equal to } —— -625

And be the Square-*Number* never ſo great, both the manner of the Operation, and the Reaſon of the Analyſis, or extraction of its Square-Root, is the very ſame.

But for the better underſtanding of all that hath been ſaid, let the Practitioner conſider well the Figure Where he may evidently ſee, how the Root of every leſſer Square being doubled, and an Unite added to it, makes up the next greater Square.

Thus twice (3) or the double Root of the next leſs Square, more by (1) being added to that Square (9) makes up the next greater Square, *viz.* (16) And twice (5) or the double Root of the next leſs Square (25) more by (1) that is to ſay (11) being added to that (25) makes it (36) which is the next greater Square.

CHAP XVII.

The way and Reaſon of extracting the Square-Root of FRACTIONS.

HE who would rightly comprehend the Nature and Reaſon of extracting the Square Root of any *Fraction*, let him conſider the *Numerator* to be a leſſer Square-*Number*, and the *Denominator* to be a greater Square-*Number*; and ſo the leſſer to be included in the greater; as for example, if it were demanded to extract the Square-Root of $\frac{9}{25}$? the *Numerator* (9) is to be conſidered as a leſſer Square-*Number*, and the *Denominator* (25) as a greater Square-*Number*, including or comprehending that leſſer or (9) And then all that is to do, is onely to extract the Square-Root of the *Numerator* (9) for a new *Numerator*, *viz.* (3) and then the Square-Root of the *Denominator* (25) for a new *Denominator*,

nominator, viz. (5) As if they were Plain *Numbers*; That is to ſay, the Square-Root of $\frac{9}{25}$ is $\frac{3}{5}$

But now, if neither the *Numerator* or *Denominator* be Pure-Square-*Numbers*, the Operation is ſomewhat more intricate, though the Reaſon of it be ſtill the ſame. As for example, if it were demanded to extract the Square-Root of $\frac{3}{5}$ ſtill the *Numerator* (3) is to be conſidered as a leſſer Square, included in a greater, *viz.* (5)

That is to ſay, The Square Root of (3) is $1\frac{2}{3}$
And the Square-Root of (5) is $2\frac{3}{5}$

Firſt, I bring the two *Fractions* $\frac{2}{3}$ and $\frac{3}{5}$ to two *Fractions* of the ſame *Denomination*, by the Rule laid down *Chap.* 8. Pag. 34. and they are $\frac{10}{15}$ and $\frac{9}{15}$. So then

The Square-Root of (3) is $1\frac{10}{15}$ or $\frac{25}{15}$
And the Square-Root of (5) is $2\frac{9}{15}$ or $\frac{39}{15}$

CHAP. XVIII.

The Vulgar method of Extracting the Cube-Root.

FOr example, let 15625 be the *Number* whoſe Root is to be extracted.

Firſt, it is to be pointed, beginning with the laſt Figure on the Right haud, and from thence to the 4th from the right hand incluſive, and ſo to the end, pointing one and leaving two. Thus 15625. Where Note that the *Number* of pricks, or points ſignify the places of the Root ſought.

Secondly, the Cube-Root is to be enquired of the laſt prickt *Number* on the left hand, (be they three, two, or one) As here the Cube-Root of 15 is to be demanded, and being found by the Table of Cube-*Numbers* Pag. 45. to be 2. I ſet it in the *Quotient* for the firſt *Number*

Number of the Cube-Root, and then ſetting the Cube of that (2) *viz.* (8) under (15) and ſubtracting the firſt out of the laſt, there remains (7) And this is the firſt ſtep, and the work ſtands thus.

$$\begin{array}{l} 1\dot{5}62\dot{5}\ (\dot{2} \\ \underline{\ \ 8} \\ \ \ 7 \end{array}$$

In the next place having ſet (6) the triple of (2 the firſt found Figure of the Root) under the laſt Figure ſave one of the next prickt *Number*, I ſquare the ſame (2) and then it is (4) and then I triple that Square (4) and make it (12) and then I ſet that (12) under the *Remainder* 762 juſt as in *Diviſion*, and this done I ask how many times, (12) is found in the Figures 76 Iuſt above them, and becauſe it may be found 5 times I ſet (5) at all adventures for the ſecond Figure of the Root, and then the work ſtands thus.

$$\begin{array}{l} 1\dot{5}62\dot{5}\ (\dot{2}\dot{5} \\ \underline{\ \ 8} \\ \ \ 7625 \\ \ \ \ \ \ \ 6 \\ \ \ 12 \\ \hline \end{array}$$

This done, I multiply (the Triple of the Root multiplied by the firſt Figure of the Root) by this ſecond Figure; That is, I multiply (12) by (5) and ſet it juſt under (76).

Again, I multiply the Square of the laſt Figure, by the Triple of the firſt, that is (25) by (6) and ſet it underneath likewiſe.

In the next place, I Cube that laſt Figure, (thatis I Cube 5) and ſet that Cube, *viz.* (125) under the ſecond Cube-*number*, that is, ſetting the Unite place of the Cube (125) under the ſecond prickt

Figure.

Figure. And add together thoſe Products, and then the work ſtands thus.

$$
\begin{array}{r|l}
1\,5\,6\,2\,5 & 2\,5 \\
8 & \\
\hline
7\,6\,2\,5 & \\
\quad 6 & \\
\hline
\quad 1\,2 & \\
\hline
\quad 6\,0 & \\
\quad 1\,5\,0 & \\
\quad\;\; 1\,2\,5 & \\
\hline
\quad 7\,6\,2\,5 &
\end{array}
$$

Laſtly, I compare this (7625) with the Remainder that was left after the deduction of the firſt *Cube-number*, (8) and finding them juſt equal, I turn it off and there remains nothing, the work is at an end, and (5) is the ſecond and the laſt Figure of the Cube Root.

But now, if the aforeſaid *Numbers* 6000, 1500, & 150. had been greater *Numbers*, and conſequently, being added together, had made a *Number* greater then (7625) then I muſt have taken a leſſer *Number*, and conſequently the ſecond Figure of the Cube-Root had been not (5) but (4).

Again, if the *Numbers* 6000, 1500 and 150. being added together, had not made a *Number* firſt equal to (7625) That is to ſay, ſuppoſe the *Cube-Number* given had been 15645, which is more by (20) conſequently after the work ended, there had remained (20) which (20) had been the *Numerator* of a *Fraction*, whoſe *Denominator* is the difference of the Cube of (25) and (26) and that *Fraction* had been ſet down thus, $\frac{20}{1951}$

And then the anſwer had been thus, the Cube-Root of (15645) is $25\frac{20}{1951}$

CHAP.

CHAP. XIX.

The Reaſon and Demonſtration of Extracting the Cube-Root.

THough the practick of this Extraction may at firſt ſight ſeem ſomething difficult, yet the Reaſon and Demonſtration of it, will I doubt not, make recompence.

And in order thereunto I ſhall firſt deſire the Practitioner to conſider the Definition of a *Cube*, as alſo the Sections of a *Cube* into its Analytical parts, according to the Rule of Perſpective.

Eucl. l. 7. *Defin.* XIX

Κύβος, &c.

A Cube *is that* Number *which is equally equal, or, which is contained under three equal* Numbers.

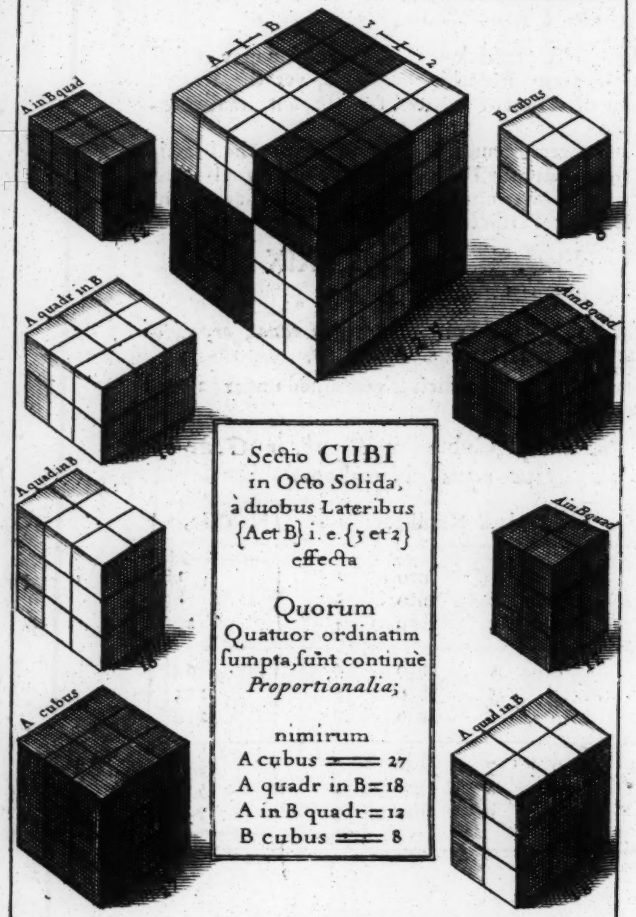
THus (8) is a *Cube* which is contained under (2) and (2) and (2).

A Table of Cubes *with their Genetive equal* Numbers.

Three equal Numbers					*Cube,*
1	into	1	into	1	1
2	into	2	into	2	8
3	into	3	into	1	27
4	into	5	into	4	64
5	into	5	into	5	125
6	into	6	into	6	216
7	into	7	into	7	347
8	into	8	into	8	512
9	into	9	into	9	729

Thus

The Section of a CUBE (125) by a *Binomial Root* (A + B) *or* (3 + 2)



Sectio CUBI
in Octo Solida,
à duobus Lateribus
{A et B} i. e. {3 et 2}
effecta

Quorum
Quatuor ordinatim
ſumpta, ſunt continuè
Proportionalia;

nimirum
A cubus = 27
A quadr in B = 18
A in B quadr = 12
B cubus = 8

Thus (3) and (3) and (3) make the Cube (27) for (3) multiplied by (3) makes the Square (9) and the Square (9) multiplied again by (3) makes the Cube (27) as is more clear by the foregoing Table.

And ſo doe (5) and (5) and (5) make the Cube (125) That is, (5) into (5) makes the Square (25) and the Square (25) into (5) makes the Cube (125) *&c.*

Wherefore to Extract a Cube-Root, is nothing elſe then to find out a *number*, which being firſt multiplied into it ſelf, and then into the Product, produces the given Cube-*Number*. Thus to extract the Cube-Root of (15625,) is to find out the *number* (25) which being firſt multiplied into its ſelf (makes 625) and then multiplied into that (625) makes the given *Number* (15625)

Now becauſe this conſtruction of the *Cube* from a ſingle Root, contributes nothing towards the finding out that Root from a given Cube-*Number*, therefore was found out by the Antients, that admirable Art of cutting or dividing the Root into two parts, which they therefore called a *Binomial Root*; and from thoſe two parts they erected 8 ſolid *numbers*, whereof the greateſt and the leaſt are always two pure Cube *numbers*, of thoſe two diſtinct parts, and of the other ſix *Parallelepipedons*, by which I mean ſolid *Numbers* made by multiplying the Square of one *Number* into another *Number*, in imitation of the Geometrical *Parallelopipedons* defined by *Euclid. lib.* 11. *Defin.* 30. to be a ſolid Figure contained under ſix Equilateral Figures, whereof thoſe which are oppoſite are Parallel.

The three greateſt *Parallelepipedons* are equal one to another, and each of them made by multiplying the Square of the greateſt part of the *Binomial-Root* into the leſſer part.

And the three leſſer *Parallelepipedons* are equal to one another, and each of them made, by multiplying the Square of the leſſer of the *Binomial* Root into the greater part.

Thus in the oppoſite Figure the whole Root is (5) and divided or cut into a greater part A—3 and a leſſer part B—2. The

The Cube of the greater A—3 is equal to——27
The Cube of the leſſer B—2 is equal to———8
One of the greater *Parallelepipedons*, or (Aq into B) is equal to———18
One of the leſſer *Parallelepipedons* or (A into Bq) is equal to———12

Then
To thoſe two Cubes, namely { The greater or Ac——27 / The leſſer or Bc——8

Adding
Three of the greater *Parallelepipedons, viz.* { (Aq) into (B) -- 18 / (Aq) into (B) -- 18 / (Aq) into (B) --- 18

And
Three of the leſſer *Parallelepipedons, viz.* { (A) into (Bq) -- 12 / (A) into (Bq) -- 12 / (A) into (Bq) -- 12

The total ſumme is the entire Cube-*Number* of the *Binomial-Root* A—3 more by B—2 that is to ſay (5) and amounteth to——125

Now the Practitioner is to conceive the Unites of the *Cube-Number* (125) to be as ſo many Dice, or Cubical Unites, and 27 of theſe being piled one upon another orderly and equally to make up (Ac) and 8 of them to make (Bc) and 18 of them to make one of the greater *Parallelepipedon's* (Aq into B) and 12 of them to make one of the leſſer Parallelepipedons (A into Bq) and then all theſe eight ſolid *Numbers* being orderly put together, to make up the entire Cube or (125) And this is the Geneſis of a Cube. *vid.* Fig.

After the very ſame manner, let the Root (25) be made *Binomial* and cut into two parts, *viz.* (20) and (5) and the greater called A, and the leſſer B

Then
The Cube of the greater A—20 is equal to---8000
The Cube of the leſſer B—5 is equal to——125
The greater *Parallelepipedon*, or (Aq into B) is equal to———2000
The leſſer *Parallelepipedon*, or (A into Bq) is equal to———500

Then

Then to these two Cubes, namely { The greater or Ac | -8000 ; The leſſer, or Bc | --125

Adding Three of the greater *Parallelepipedons*, *viz.* { (Aq) into (B) 2000 ; (Aq) into (B) 2000 ; (Aq) into (B) 2000

And Three of the leſſer *Parallelepipedons*, *viz.* { (A) into (Bq) 500 ; (A) into (Bq) 500 ; (A) into (Bq) 500

Summe —1 5 6 2 5

The total Summe of all the eight Solids, is the Summe of the entire Cube, which amounteth to —— ----1 5 6 2 5

All which is conſonant to that *Theorem* of *Ramus*, (which is in imitation of that of *Euclid* concerning a Square *Number*.

P. R A M I *Geometr.*

*Lib. XXIIII, De Cubo pag.*135.

IF a right Line be cut into two Segments, the Cube of the whole ſhall be equal to the Cubes of the Segments, and thrice the double Solid made of the Square of one Segment.

Thus in the laſt Example, the Square of the greater Segement, multiplied by the leſſer, and the Square of the leſſer Segment multiplied by the greater Segment, (for that is meant by the double Solid made of the Square of the one Segment into the other Segment) is (2500)

And, The triple of that (2500) is———7500
To which adding the two Cubes, or——8125

The total is the entire Cube, *viz.* —— 15625

After all this, it will be eaſie for the meaneſt capacity to conceive, that whilſt by the vulgar Rules of Extracting the Cube-Root of 15625, the firſt Cube of (8) is ſubtracted out of (15) it is indeed (8000) or (Ac) that is the Cube of the greater Segment ſubtracted out of (15625) whoſe Cube-Root is

(2 or

(2) or in appearance, but in reality is (20) or the greater Segment (A)

Again, whereas in the next place the vulgar Rule directs, to ſet down the triple of the firſt found Figure (2) under the laſt Figure ſave one of the next *Cube-Number, viz.* (2) and then to ſquare that (2) and ſo make it (4) and then to triple that (4) and ſo make it (12). And then to find how many times that (12) can be found in the remainder, or (7625) (which ſuppoſe to be 5 times) with this proviſion, that after (12) has been deducted thus 5 times out of (7625) there ſtill remain a *Number*, either greater or equal to the Square of that (5), that is (25) multiplied by the triple of the firſt-found Root (2) more by the Cube of 5) that is (125) and if ſo, the work ſtands good, and the Cube Root is (25).

The true meaning of all is this.

1. The firſt found Root (2) is Really (20) and is the greater Segment (A)
2. The Square of that (20) is (400) and is (A) Square.
3. The Triple of that (400) is 1200) and is (A) Square thrice.
4. This (1200) is found in (7625) 5 times, which (5) is the leſſer Segment (B)
5. This (1200) multiplied by (5) or (B) makes (6000) and is thrice (A) Square into (B)
6. The Square of that 5) is (25 or (B) Square.
7. This (25) or (B) Square multipled by (6) or the triple of (A) makes (1500) and is (A) into (B) Square thrice.
8. The Cube of this (5) is (125) or (B) Cube.

All theſe being added together, makes up the entire Cube.

That is to ſay,

A, Cube, or the Cube of the greater Segment---8000

2. A Square into B, or the triple of the ſolid *Number* made of the Square of the greater Segment multiplied by the leſſer Segment ---6000

3. A, into B Square, or the triple of the solid *Number* made of the Square of the lesser Segment multiplied by the greater Segment ——— 1500

B Cube, or the Cube of the lesser Segment — 125

The total sum is the intire Cube ——— 15625

The given Cube *Num.* (AB the *Binomial* 25 Root, *viz.* A = 20)

15	625	
8	—	—(A) Cube.
7	625	Remainder
	60	(A) thrice
1	200	(A) Square thrice
6	000	(A) Square into (B) thrice, or the triple of the greater Parallel.
1	500	(A) into (B) Square thrice, or the triple of the lesser Parallel.
	125	(B) Cube.
7	625	

Aggregate of the three former sums and equal to the Remainder abovesaid.

CHAP. XX.

A plain and easie Method of extracting the Square-Root of any Number, (how great soever) without the help of either Multiplication, or Division.

For Example.

Let the Square number be———

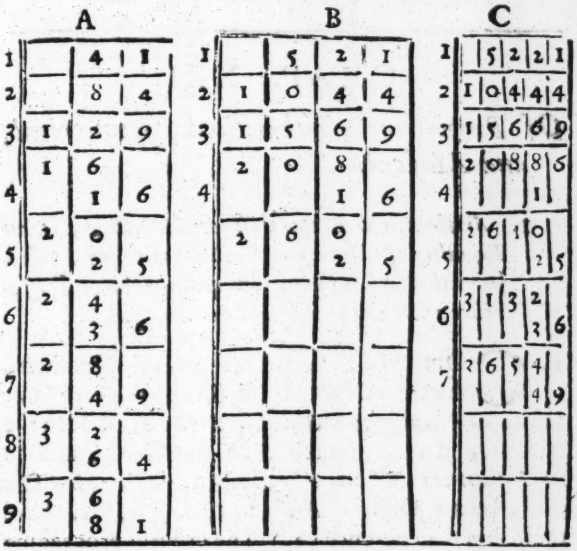
2	6	1	6
6	84	34	56
4			
2	84		
2	76		
	8	34	
	5	21	
	3	13	56
	3	13	56
	0	00	00

THere are therefore 4 Figures in the Root.

The first, *viz.* (2) is found by inspection.

The second is had by the *Tariffa* (A) thus,

Take the double of the first Figure (2) *viz.* 4, and make a *Multiplicaton* Table of it, mingling with the respective Products, the Squares of the 9 Digits as in *A*. So shall you by inspection find that 276 being the neareſt Number to 284, stands over againſt 6 in the Margint, and therefore (6) is the second Figure of the Root.



In the next place, I offer to make the *Tariffa* B, for the double of the two firſt Figures 2 and 6, *viz.* 52. But by inſpection, I find that it is needleſs to fill up the ſpaces, the very ſecond Product being too great for the Number 834. Therefore (1) muſt be the next Figure of the Root.

Laſtly, I make a *Tariffa* for the double of the firſt 3 Figures of the Root, 2, 6, and 1, *viz.* 522. And by that time I come to the ſeventh place, I find that 6 is the laſt Figure, for it gives me 31356, which is the very Number I ſought for.

This Method is the moſt certain, and has the leaſt of difficulty in it of any Methods I ever yet ſaw.

CHAP.

CHAP. XXI.

Of Proportions Arithmetical, Geometrical, and Muſical.

1. *Arithmetical Proportion or Habitude, is an Equality of Differences, That is to ſay, when ſeveral Numbers have one and the ſame Difference: And this Habitude is two-fold.*

1. CONtinued: When of ſeveral Numbers, the ſecond exceeds the firſt by the very ſame Unity or Number of Unities, as the third doth the ſecond, and as the fourth doth the third; and ſo *in Infinitum*, Thus, 1, 2, 3, 4, 5, 6, 7, 8, *&c.* differ by an Unit. And 1, 3, 5, 7, 9, *&c.* have their equal Difference (2.) The orderly proceeding of which Numbers from the leſſer to the greater in a Scalal-way, is that which is properly called *Arithmetical Progreſſion.*

2. Disjunct, when the ſecond exceeds the firſt by the ſame Number of Units, as the fourth doth the third, but not as the third doth the ſecond. *As for Example,* 1, 3, 7, 9, are four Disjunct *Arithmetical Proportionals.* For (3) exceeds (1) by the ſame Number of Units as 9) doth (7), but not as (7) doth (3). And thus, 2, 7, 10, 15, are four Proportionals of the ſame kind, for (7) exceeds (2) by 5, Units, and ſo doth (15) exceed (10).

2. Geo-

2. *Geometrical Proportion or Habitude, is the Equality of* Ratio's, *that is to ſay, it is that which ſhews what part or parts one Number is of another.*

Thus, (1, 2, 4, 8) (2, 4, 8, 16,) (3, 6, 12, 24) (4, 8, 16, 32, 64) are *Geometrical Proportionals:* For in the firſt Example, as (1) is the half of (2), ſo is (2) the half of (4), and again, (4) the half of (8), as may be ſeen in the following Tables.

3. *Muſical Proportion or Habitude, is when the firſt Number hath the ſame Proportion to the third, which the Difference between the firſt and the ſecond hath to the Difference between the ſecond and the third.*

As in (3, 4, 6), (3) is the half of (6), and ſo is (1) or the Difference between (3) and (4) the half of (2), or the Difference between (4) and (6); and ſo in (6, 8, 12). But of this in its proper place.

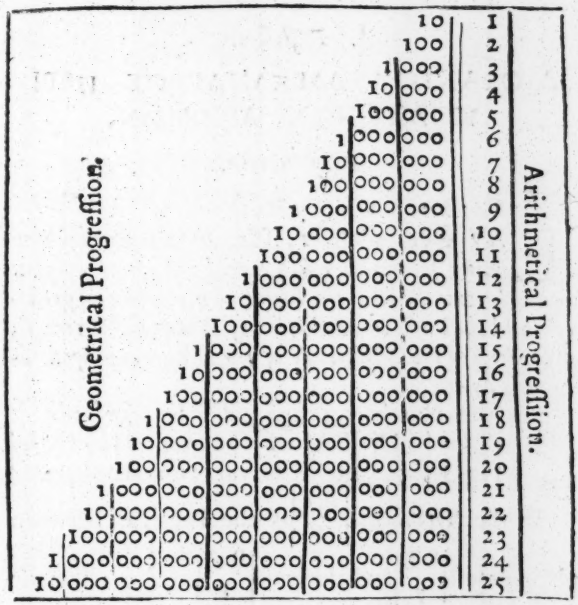
That which is at preſent to be handled, is the Nature, Properties, and Similitude of the two firſt kinds of Proportion, Namely *Arithmetical* and *Geometrical*, which may be viewed in their ſeveral *Progreſſions*, by the following Tables.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144
13	26	39	52	65	78	91	104	117	130	143	156
14	28	42	56	70	84	98	112	126	140	154	168
15	30	45	60	75	90	105	120	135	150	165	180
16	32	48	64	80	96	112	128	144	160	176	192
17	34	51	68	85	102	119	136	153	170	187	204
18	36	54	72	90	108	126	144	162	180	198	216
19	38	57	76	95	114	133	152	171	190	209	228
20	40	60	80	100	120	140	160	180	200	220	240
21	42	63	84	105	126	147	168	189	210	231	252
22	44	66	88	110	132	154	176	198	220	242	264
23	46	69	92	115	138	161	184	207	230	253	276
24	48	72	96	120	144	168	192	216	240	264	288
25	50	75	100	125	150	175	200	225	250	275	300
26	52	78	104	130	156	182	208	234	260	286	312
27	54	81	108	135	162	189	216	243	270	297	324
28	56	84	112	140	168	196	224	252	280	308	336
29	58	87	116	145	174	203	232	261	290	319	348
30	60	90	120	150	180	210	240	270	300	330	360
31	62	93	124	155	186	217	248	279	310	341	372
32	64	96	128	160	192	224	256	288	320	352	384
33	66	99	132	165	198	231	264	297	330	363	396
34	68	102	136	170	204	238	272	306	340	374	408
35	70	105	140	175	210	245	280	315	350	385	420
36	72	108	144	180	216	252	288	324	360	396	432
37	74	111	148	185	222	259	296	333	370	407	444
38	76	114	152	190	228	266	304	342	380	418	456
39	78	117	156	195	234	273	312	351	390	429	468
40	80	120	160	200	240	280	320	360	400	440	480

1	1	1
2	3	4
4	9	16
8	27	64
16	81	256
32	243	1024
64	729	4096
128	2187	16384
256	6561	65536
512	19683	262144
1	1	1
5	6	7
25	36	49
125	216	343
625	1296	2401
3125	7756	16807
15625	46536	117649
68125	279216	823543
340625	1675296	5764801
1703125	10051776	40353607
1	1	1
8	9	10
64	81	100
512	729	1000
4096	6561	10000
32768	59049	100000
262144	531441	1000000
2097152	4782969	10000000
16777216	43046721	100000000
134217728	387420489	1000000000
1	1	1
11	12	13
121	144	169
1331	1728	2197
14641	20736	28561
161051	248832	371293
1771561	2985984	4826809
19487171	35831808	62748517
214358881	429981696	815730721
2357947691	5159780352	10604499373

Geometrical Progression.	Arithmetical Progression.
1	1
3	3
9	5
27	7
81	9
243	11
729	13
2187	15
6561	17
19683	19
59049	21
177147	23
531441	25
1594323	27
4782969	29
14348907	31
43046721	33
129140163	35
387420489	37
1162261467	39
3486784401	41
10460353203	43
31381059609	45
94143178827	47
282429536481	49
847288609443	51
2541865828329	53
7625597484987	55
22876792454961	57
68630377364883	59
205891132094649	61
617673396283947	63
1853020188851841	65
5559060566555523	67
16677181699666569	69
50031545098999707	71

Geometrical Progression.	Arithmetica. Progression.
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	22
4194304	23
8388608	24
16777216	25
33554432	26
67108864	27
134217728	28
268435456	29



In this *Progreſſion* it is more viſible then in any other, how *Addition* and *Subſtraction* in *Arithmetical Progreſſion* anſwers to *Multiplication* and *Diviſion* in *Geometrical Progreſſion.* For, as in *Geometrical Progreſſion*, 1000 Multiplied by 100,000 produce 100,000,000; So in *Arithmetical Progreſſion*, the Number anſwering to 1000, and 100,000. *viz.* (3) and (5) being added together make (8), which anſwers to the Product 100,000,000. Again, as in *Geometrical Progreſſion* (100,000 000) being divided by (100,000) the Quotient is (1000.) So in *Arithmetical Progreſſion*, it from the greateſt of their Coreſpondent Numbers, *viz.* (8) you Subſtract any one of the other, *viz.* (5,) the remainder is (3) and anſwereth to (1000.)

Arith-

I. *Reflection.*

ΔΙΟΦΑΝΤΟΥ ΑΛΕΞΑΝΔΡΕΩΣ ΠΕΡΙ ΠΟΛΙΓΟΝΩΝ ΑΡΙΘΜΩΝ.

Prop. Tertia.

β. α.. γ.. δ.. ε. B. A.. G.. D.. E.

Ἐὰν ὦσιν ἀριθμοὶ, &c. *If Numbers (how many ſoever they be) exceed one another by an equal Interval, then the Interval between the greateſt, and the leaſt, is Multiplex of that equal Interval, according to the Multitude of the Numbers propounded, leſs by one.*

Let the Numbers propounded be four, *viz.* $\left|\begin{smallmatrix}B\,A\\1\end{smallmatrix}\right|$ $\left|\begin{smallmatrix}B\,G\\3\end{smallmatrix}\right|$ $\left|\begin{smallmatrix}B\,D\\5\end{smallmatrix}\right|$ $\left|\begin{smallmatrix}B\,E\\7\end{smallmatrix}\right|$ whoſe common Interval is equal to $\left|\begin{smallmatrix}A\,G\\2\end{smallmatrix}\right|$

By the *Hypotheſis* $\left|\begin{smallmatrix}A\,E\\6\end{smallmatrix}\right|$ is the Interval between the greateſt $\left|\begin{smallmatrix}B\,E\\7\end{smallmatrix}\right|$ and the leaſt $\left|\begin{smallmatrix}B\,A\\1\end{smallmatrix}\right|$: And likewiſe the three Numbers $\left|\begin{smallmatrix}A\,G\\2\end{smallmatrix}\right|$ $\left|\begin{smallmatrix}G\,D\\2\end{smallmatrix}\right|$ $\left|\begin{smallmatrix}D\,E\\2\end{smallmatrix}\right|$ and every of them equal to the Common Difference, and equal the one to the other. And the multitude of them equal to the Multitude of the Numbers given leſs by one (*viz.* the leaſt.) And laſtly, the Aggregate of theſe three Numbers is equal to the Interval $\left|\begin{smallmatrix}A\,E\\6\end{smallmatrix}\right|$ (the parts united being equal to the whole.)

That is to ſay,

$\left|\begin{smallmatrix}A\,E\\6\end{smallmatrix}\right|$ or the Interval between the greateſt $\left|\begin{smallmatrix}B\,E\\7\end{smallmatrix}\right|$ and the leaſt $\left|\begin{smallmatrix}B\,A\\1\end{smallmatrix}\right|$ is Multiplex of the common Interval

A G

I. *Reflection.*

IF Numbers (how many ſoever they be) contain the one the other by an equal Ratio; *Then the greateſt of thoſe Numbers is Multiplex of the Powers of the Denomination of that equal* Ratio *Multiplyed by the leaſt, according to the Multitude of the Numbers given, leſs by one*

This Reflection I have framed for *Geometrical Proportionals*, in imitation of that *Diophantus* for *Arithmetical Proportionals*.

Let the Numbers given be Four, *viz.* 2, 6, 18, 54.

And let the Denominator of the *Ratio* be 3.

Then by the *Hypotheſis*, the firſt Multiplyed by (3) is equal to the ſecond; and the ſecond Multiplyed by (3) is equal to the third; and the third Multiplyed by (3) is equal to the fourth: And ſo *in Infinitum*.

That is to ſay,

1. The firſt Term (2) is equal to 2 — 1ſt power of the ſingle Ratio, or the Ratio
2. The 2d Term (6) is equal to 2 into 3 — 2d power of the Ratio, or the Ratio Squared.
3. The 3d Term (18) is equal to 2 into 3 into 3 — 3d power of the Ratio, or the Ratio Cubed.
4. The 4th Term (54) is equal to 2 into 3 into 3 into 3

That is to ſay, in a *Symbolical* way,

Let there be any Number of Proportional Numbers, as, A, B, C, D, E, F, &c. And the Denomination of the *Ratio* be R.

A = A

$\left|\begin{matrix} A & G \\ & 2 \end{matrix}\right|$ according to the Multitude of the Numbers propounded leſs by one, Which was to be Demonſtrated.

In a *Symbolical* way, Thus,

Then let the Terms be A, B, C, D, and the Common Difference E.

Thus,	Or thus,
A=A	A=A
B=A+E	B=A+1E
C=A+E+E	C=A+2E
D=A+E+E+E	D=A+3E

In plain *Engliſh* thus,

1ſt Term (1) is equal to 1
2d Term (3) is equal to 1 more by 2.
3d Term (5) is equal to 1 more by 2 more by 2.
4th Term (7) is equal to 1 more by 2 more by 2 more by 2.

That is to ſay, by the *Hypotheſis*, the greateſt Term is equal to the leaſt, and as many Differences as there are more Terms beſides the leaſt; therefore the greateſt Term leſs by the leaſt is Multiplex of the Difference according to the Number of Terms leſs by one.

As was to be Demonſtrated.

A ⊏ A
B ⊏ A into R
C ⊏ A into R into R
D ⊏ A into R into R into R
E ⊏ A into R into R into R into R
F ⊏ A into R into R into R into R into R
G ⊏ A into R into R into R into R into R into R

Or thus,

		Powers of R.
A	⊏ A	
B	⊏ A into R	1
C	⊏ A into R Square	2
D	⊏ A into R Cube	3
E	⊏ A into R Squared Square	4
F	⊏ A into R Squared Cube	5
G	⊏ A into R Cubick Cube	6

As this Reflection carryes its evidence along with it, ſo is it of admirable Speculation and Uſe, as hereafter will appear.

Geome-

II. *Reflection.*

WHere *three Numbers are three* Arithmetical Proportionals, *the Sum of the two Extreams is equal to the Double of the Mean.*

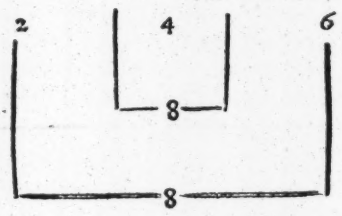
Let the three Numbers be (2, 4, 6)
And the Common Difference be (2)

Then by the Firſt Reflection.
The Firſt Term is 2
The ſecond Term is 2 more by the Difference once.
The third Term is 2 more by the Difference twice.

Whence it is evident and obvious, That twice (2) more by twice the Difference, is both the ſum of the firſt and third Terms, and alſo the double of the ſecond or Mean.

Which was to be Demonſtrated.

Vid. Fig.



Arith-

II. *Reflection.*

Eucl. Lib. 7. Prop. 20.

Ἐὰν τρεῖς ἀριθμοὶ, &c. *If there be three Numbers in proportion, the Number contained under the Extreams, is equal to the Square made of the Mean; And if the Number contained under the Extreams be equal to the Square of the Mean, thoſe three Numbers ſhall be in Proportion.*

Let the three Numbers be (2, 6, 18)
And the Equal *Ratio* (3.)
Then by the firſt Reflection.

The firſt Term is 2	A
The ſecond Term is 2 into (3)	A×R
The third Term is 2 into (the Square of 3)	A×R×R

Whence it is evident and obvious in the firſt place, that the firſt Term drawn into the third is equal to 2 into 2, into the Square of 3.

Again it is evident, That the Square of the ſecond Term is equal to the Square of 2 into the Square of 3.

But theſe two are equal; That is to ſay,

(2) into (2) into (the Square of 3) = 36
(the Square of 2) into (the Square of 3) = 36

Ergo,

The Product of the Firſt Multiplyed by the Third, is equal to the Square of the Second or Mean. Which was to be demonſtrated. *Vid. Fig.*

Geome-

III. *Reflection.*

WHere *four Numbers are four* Arithmetical Proportionals, *the Sum of the firſt and fourth, is equal to the Sum of the ſecond and third.*

Firſt, let them be four continued Proportionals, *viz.* (4, 12, 20, 28)
And their Common Difference (8)

Then by the ſecond Reflection.

The firſt Term is 4
The 2d Term is 4 more by once the Difference.
The 3d Term is 4 more by twice the Difference.
The 4th Term is 4 more by thrice the Difference.

Secondly, let them be four diſcontinued Proportionals, *viz.* (4, 12, 30, 38.)
And their Common Difference (8)

Then by the ſecond Reflection, and the *Hypotheſis*.

The firſt Term is 4
The 2d Term is 4 more by once the Difference.
The 3d Term is 30
The 4th Term is 30 more by once the Difference.

III. *Reflection.*

Eucl. Lib. 7. Prop. 19.

ΕΑν τέσσαρες, &c. *If there be four Numbers in proportion, the Number produced of the first and fourth, is Equal to the Number which is produced of the ſecond and third. And if the Number produced of the firſt and fourth be Equal to that produced of the ſecond and third: Thoſe four Numbers ſhall be in proportion.*

Firſt let them be four continued Proportionals. *viz.* (2, 6, 18, 54) and the Equal *Ratio* (3).

Then by the Second Reflection.
The firſt Term is 2.
The ſecond Term is 2 into (3).
The third Term is 2 into the Square of (3).
The fourth Term is 2 into the Cube of (3).

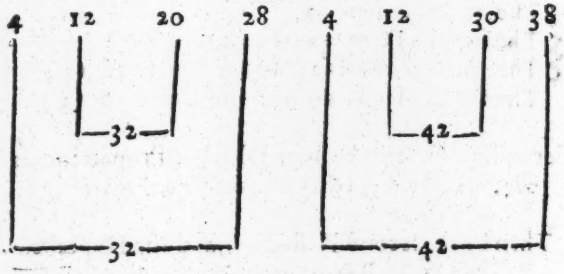
Secondly, let them be four Disjoynt Proportionals. *viz.* (2, 6, 54, 162) and the Equal *Ratio* (3).

Then by the ſecond Reflection, and the Hypotheſis.
The firſt Term is 2.
The ſecond Term is 2 into (3).
The third Term is 54.
The fourth Term is 54 into (3).

In the first Example it is evident,
That twice(4)more by thrice the Difference(24), is both the Sum of the Extreams, and also the Sum of the second and third, *viz.* 32.

In the second Example it is as evident,
That (4) more by (30) more by once the Difference, is both the Sum of the Extreams, and also of the second and third Terms, *viz.* 42.

Vid. Fig.



4. *Reflection.*

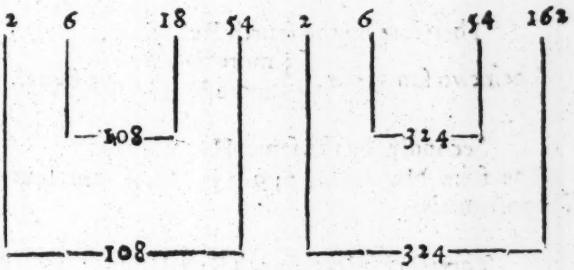
In the firſt Example it is evident by the Multiplication of Powers.

That the Square of (2) Multiplyed by the Cube of (3) is both the Product of the Extreams, and alſo of the two Means.

In the ſecond Example it is as evident,

That (2 into 3) Multiplyed by (54) is both the Product of the Extreams, and alſo of the Means.

Vid. Fig.



IV. *Reflection.*

IN all Continued Arithmetical Progreſſions, (how many ſoever the Terms be) the Sum of the Extreams is equal to the Sum of any two of the other Terms equidiſtant from the Extreams, and to the double of the Middle Term, in caſe the Number of Terms be odd.

Let the Number of Terms be Seven.
viz. 3, 6, 9, 12, 15, 18, 21, and the Common Difference (3).

First by the Hypotheſis.
The four Numbers (3, 6, 18, 21) are four Proportionals.

Therefore by the fourth Reflection.
Their two ſums, *viz.* { 3 more by 21 / 6 more by 18 } are Equal.

Secondly, by the ſame Hypotheſis.
The four Numbers (6, 9, 15, 18,) are four Proportionals.

Therefore by the fourth Reflection.
Their two ſums, *viz.* { 6 more by 18 / 9 more by 15 } are Equal.

Laſtly, by the ſame Hypotheſis.
The three Numbers (9, 12, 15) are three Proportionals.

There-

IV. *Reflection.*

IN all Continued Geometrical Progreſſions (how many ſoever the Terms be) the Product of the Extreams is equal to the Product of any two of the other Terms equidiſtant from the Extreams, and to the Square of the Middle Term, in caſe the Number of Terms be odd.

Let the Number of Terms be Seven.

Viz. (2, 6, 18, 54, 162, 486, 1458); and the equal *Ratio* (3).

Firſt, by the Hypotheſis.

The four Numbers (2, 6, 486, 1458) are four Proportionals.

Therefore by the fourth Reflection.

Their two Products, *viz.* { 2 into 1458 / 6 into 486 } are Equal.

Secondly, by the ſame Hypotheſis.

The four Numbers (6, 18, 162, 486) are four Proportionals.

Therefore by the fourth Reflection.

Their two Products, *viz.* { 6 into 486 / 18 into 162 } are Equal.

Laſtly, by the ſame Hypotheſis.

The Three Numbers (18, 54, 162) are three Proportionals.

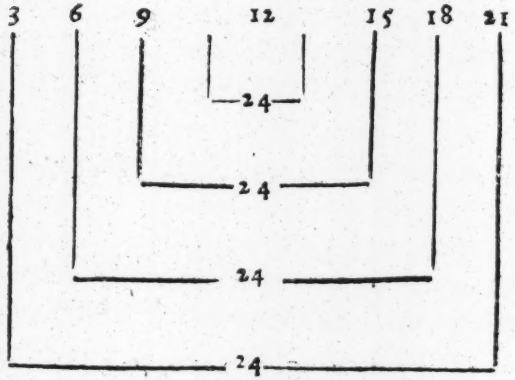
Therefore by the third Reflection.

{9 more by 15 } are Equal.
{12 doubled }

Therefore by the 1*ſt* Ax. *Euclid.*

{3 more by 21 }
{6 more by 18 } are all Equal one to a-
{9 more by 15 } nother.
{12 doubled } As was to be Demonſtrated.

Vid. Fig.



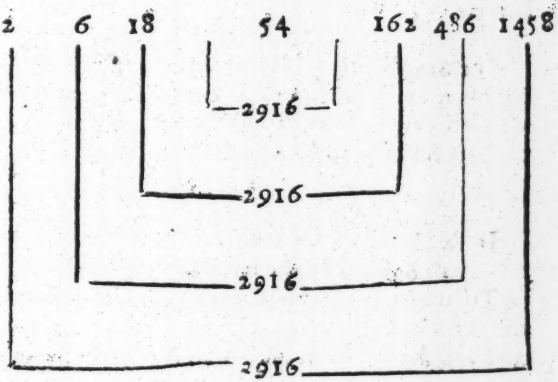
Therefore by the third Reflection.

18 into 162 and 54 Squared, are Equal.

Wherefore,

2 into 1458, 6 into 486, 18 into 162, 54 Squared, do make 2916; and consequently are all Equal one to another: Which was to be Demonstrated.

Vid. Fig.



The foregoing Reflections applyed to the *Golden Rule*, or *Rule of Three.*

The *Rule of Three*, or the *Golden Rule.*

THis Rule is either Single, or Compound;

1. *The Single Rule of Three.*

The *Single Rule of Three*, is when three Numbers are Given, and a fourth Proportional Demanded.

Now this Single Rule, is either Direct, or Inverse.

1. *Direct.*

The *Single Rule of Three Direct*, is when three Numbers are Given, and a fourth is Demanded, which bears the same proportion to the Third, as the Second bears to the First.

Example.

If 4 Acres of Ground cost 80 Pound, what will 8 Acres of the same Ground cost?

To understand this Operation, look back to the third Reflection upon *Geometrical Proportion*, and you shall find it Demonstrated, that if there be four Proportionals, the Product of the First and Fourth is equal to the Product of the Second and Third.

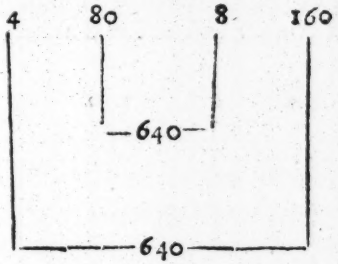
Wherefore in this *Example.*

If the Product of the Second and Third Term, *viz.*

viz. (640) be Divided by the Firſt, namely (4) The Quotient, *viz.* (160) is the fourth Proportional ſought.

And for a Proof of this,

Let the fourth found Term (160) being Multiplyed by the firſt (4); the Product will be the ſame with the Product of the third Multiplyed by the ſecond, *viz.* 640.



The Argumentation is plainly thus,

If the Product of (80 by 8) be equal to the Product of (4 by the unknown Number.)

Then,

The Quotient will be the very ſame, whether I Divide the Product of (80 by 8,) or whether I Divide the Product of (4 by the unknown Numb.r) by 4. For either of the Products being 640, the Quotient muſt needs be 160.

But now it is manifeſt, that if

I Multiply 4 by 160, and Divide the Product back again by 4, it will give 160 for the Quotient. Becauſe, whatever Multiplication doth, is again undone by Diviſion. And this is the true and genuine Reaſon of the Operation in this *Rule of Three.*

That

That is,

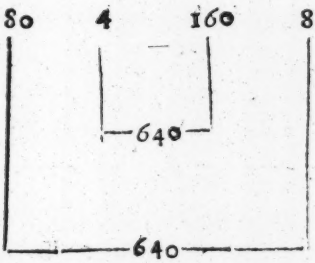
As (4) is found in (80) juſt 20 times, ſo is (8) found in (160) juſt 20 times.

Or thus,

As 4 Multiplyed by 20 makes 80, ſo 8 Multiplyed by 20 makes 160.

2. *Example.*

If 80 *l.* will buy 4 Acres, what will 160 *l.* buy? The Anſwer will be found as before.



By Dividing the Product of (4 into 160) *viz.* (640) by (80) For then the Quotient will be 8.

For as 80 contains 4, twenty times, ſo 160 being Divided by 20, the Quotient is 8.

2. *Inverſe.*

The *Single Rule of Three Inverſe*, is when there are three Numbers given, and a fourth demanded, which bears the ſame Proportion to the ſecond, as the third doth to the firſt.

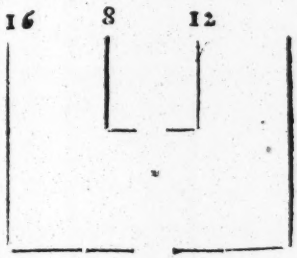
Example.

If a quantity of Hey will keep 8 Horſes 12 Days, How many Days will the ſame quantity keep 16 Horſes.

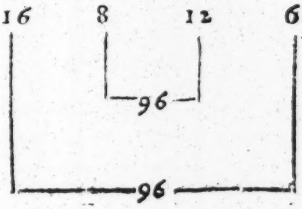
Here

Here it is obſervable, that moſt of thoſe who have hitherto Treated of this *Rule of Three*, puzzle both young Learners and themſelves, with this diſtinction of *Direct* and *Inverſe*, as though they were two diſtinct and different things, whereas in truth they are the very ſame thing, only care muſt be taken how to place them. For the true unfolding of this Queſtion, is this,

Look what proportion 16 Horſes bear to 8 Horſes, ſo do 12 Days bear to a fourth Number of Days. And they ought to be placed thus,



And then the operation is the very ſame as before; for the Product of 8 by 12 is (96) which being Divided by 16, the Quotient is (6), which is the fourth Number ſought.



2. *The*

2. *The Double Rule of Three.*

And this is { 1. *Direct.* 2. *Inverse.*

1. *Direct.*

THe *Double Rule of Three* is when more then three Terms are given; as the ordinary Books of Arithmetick teach us.

1. *Example.*

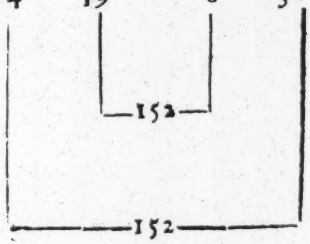
If 4 Men ſpend 19 Pound in three Months, how many Pounds will 8 Men ſpend in nine Months?

The Reſolution of this Queſtion is thus performed.

1. If four Men ſpend 19. what will 8 Men ſpend.

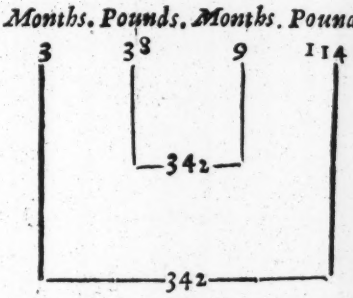
Men. *Pounds.* *Men.* *Pounds.*

4 19 8 38



2. If

2. If 38 *l.* be ſpent by any Number of Men in 3 Months, how many Pounds will be ſpent by ſuch a Number of Men in 9 Months?

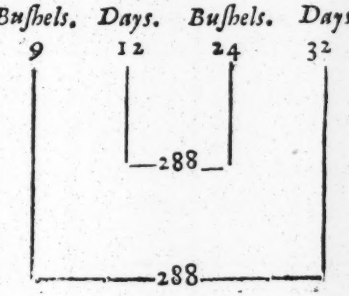


2. *Example.*

If 9 Buſhels of Oats ſerve 8 Horſes 12 Days, how many Days will 24 Buſhels laſt 16 Horſes.

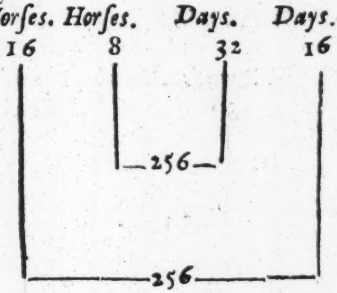
Read it thus,

1. If 9 Buſhels laſt 12 Days, how many Days will 24 Buſhels laſt?



2. If

2. As 16 Horſes is to 8 Horſes, ſo is 32 Days to 16 Days.



And this is the whole Intrigue of the *Golden Rule*, or the *Rule of Three.* The which being rightly underſtood, (together with the foregoing Reflections on *Arithmetical* and *Geometrical* Proportion) thoſe other Rules of *Fellowſhip*, and *Alligation*, as likewiſe the *Rule of Falſe*, will not be at all difficult. And therefore I think it needleſs to multiply Examples.

2982
852
1278

139302

1962
327
1962

The Diameter *of any* Circle *being given in* Integers, *to find the* Peripherie, *and the* Square-Root *of the* Area, in Infinitum, *without the help of either* Multiplication, Diviſion, *or* Extraction *of the* Root.

FOr Example, Let the given *Diameter* of a *Circle* be 351. And let it be required to find the *Peripherie* (or *Circumference.*) Firſt I ſet down (300,) under that (50,) and beneath that 1, as in the following Operation. This done, I firſt ſeek the *Peripherie* of a *Diameter* of (3) in the ſubſequent *Table*, viz. 9,42477795, and becauſe it is the place of *Hundreds*, I add thereto on the right hand two *Ciphers* (00.) Then, under that Sum I ſet the Number of the *Peripherie* of (5.) adding to it one *Cypher*, becauſe it is the place of *Tens* (viz. for 50.) Laſtly, I place under the two Sums aboveſaid the *Peripherie* of (1.) without adding any *Cypher*, becauſe it is the place of *Units*. And adding theſe 3 Sums together, (evermore diſtinguiſhing the *Integers* from the *Fractions*, after the 8th place from the right hand, with a *Comma*, or *Line* of *Seperation*) the *Sum*, or *Aggregate* (viz. 1102, 6990215,) is the true *Peripherie* of that *Circle*, whoſe *Diameter* is 351.

300—— 942	4777,9500
50—— 157	0796,3250
1—— 3	1415,9265
351 *ſums* 1102	6990.2015

After the very ſame manner, if it were required to give the *Square-root* of the *Area* of that very *Circle*, whoſe *Diameter* is 351. I add the 3 following *Sums*, and the *Aggregate* is what I deſire. *For Example.*

300——265	8680,7700
50—— 44	3113,4620
1—— 0	8862,2692
351 *ſum* 311	0656,5012

If the firſt place of the given *Diameter* be of *Thouſands*, there muſt be 3 *Cyphers*; if of *Tens* of *Thouſands*, there muſt 4 *Cyphers* be added on the right hand, and ſo *in Infinitum*.

Diamet.	*Peripherie.*	*Square-root of the* Area
1	3 1415,9265	8862,2692
2	6 2831,8530	1 7724,5385
3	9 4247,7795	2 6586,8077
4	12 5663,7060	3 5449,0770
5	15 7079,6325	4 4311,3462
6	18 8495,5590	5 3173,6155
7	21 9911,4855	6 2035,8847
8	25 1327,4120	7 0898,1540
9	28 2743,3385	7 9760,4232

But in case of *Fractions*, as for *Example*, if the *Diameter* had been given $351\frac{1}{4}$, and the *Peripherie* were demanded: The way is, to take $\frac{1}{4}$ of 3, 1415,9265. (*viz.* 0,7853,9816) and to add it to 1102,6990,2015, which makes 1103,4844,1831 for the *Peripherie* of that *Circle* whose *Diameter* is is $351\frac{1}{4}$. And the same method is to be used for any *Fraction* whatsoever.

The Circumference of a Circle being given to find the Diameter, and the Square-root of the Area, in Infinitum, *without the help of either Multiplication, Division, or Extraction of the Root.*

Circum.	*Diameter.*	*Square-root of the* Area
1	3183,0988	2820,9479
2	6366,1977	5641,8958
3	9549,2965	8462,8437
4	1 2732,3954	1 1283,7916
5	1 5915,4943	1 4104,7395
6	1 9098,5931	1 6925,6875
7	2 2281,6920	1 9746,6354
8	2 5464,7908	2 2567,5833
9	2 8643,8897	2 5388,5312

This Table is to be used after the very same manner with the former.

A

A Table of the Fixed Feaſts, and other Solemn Days to be obſerved in the Church of England.

All Sundays.

New-years-day, or *Circumciſion*	*Jan.* 1
Twelf day, or *Epiphany*	*Jan.* 5
Martyrdom of K. *Charles* I.	*Jan.* 30
Purification of the Virgin *Mary*	*Febru.* 2
LADY-DAY, or the Annuntiation of the Virgin *Mary*	*Mar.* 25
Mark Evang.	*April* 25
May day, or *Phillip and Jacob*	*May* 1
Birth and return of *Charles* II.	*May* 29
MIDSUMMER or *John Baptiſt.*	*June* 24
James Apoſtle	*July* 25
Bartholomew Apoſtle	*Aug.* 24
Matthew Apoſtle	*Sept.* 21
MICHAELMAS, or *Mich. Archangel*	*Sept.* 29
Luke the *Evangeliſt*	*Octo.* 18
Simon and *Jude*	*Octo.* 28
All Saints	*Nov.* 1
Powder Treaſon	*Nov.* 5
Andrew Apoſtle	*Nov.* 30
Thomas Apoſtle	*Dec.* 21
CHRISTMAS, or *Birth of our Lord*	*Dec.* 25
St. *Stephen*	*Dec.* 26
St. *John Evang.*	*Dec.* 27
Innocents	*Dec.* 28

Other Remarkable Days.

Valentine	*Febr.* 14
Equal day and night	*Mar.* 10
St. *George*	*April* 23
Longeſt day, or *Barneby*	*June* 11
Swithin	*July* 15
Lammas	*Augu.* 2
Equal day and night	*Sept.* 12
Shorteſt day	*Dece.* 11

HILLARY TERM.

Begins	*Jan.* 23
Ends	*Febr.* 12
and hath Four Returns.	
1. Octab. Hil.	*Jan.* 20
2. Quind. Hil.	*Jan.* 29
3. Craſt. Pur.	*Febr.* 3
4. Octab. Pur.	*Febr.* 10

MICHAELMAS TERM

Begins	*Octob.* 23
Ends	*Nov.* 28
and hath Six Returns.	
1. Tres Mich.	*Octo.* 21
2. Menſ. Mich.	*Octo.* 29
3. Craſt. An.	*Nov.* 4
4. Craſt. Mar.	*Nov.* 11
5 Oct. Mar.	*Nov.* 18
6. Quin. Mar.	*Nov.* 27

The Uſe of this Table.

1. SEek the name of the Month in the left-hand Margent, and guiding your Eye to (1) in the Table, find out the day of the Month, which you ſhall find either above, or beneath in that Column.

2. From that day of the Month, guide your eye back to the Number in the left-hand Margent that ſtands againſt it in the Column under the Title (*Day of the Month.*)

3. From that Number guide your Eye to (1) in the Table, and in that Column find the Number that is the Number of the Prime for that Year, and from thence guide your Eye to the right-hand Margent, ſo have you your deſire.

Example.

The Tenth day of *May*, 1665. I deſire to know what Sign the Moon is, *&c.*

1. The Prime for that Year is 13.

2. I find *May* in the left-hand Margent, and guiding my Eye to (1) in the Table, and in that Column to 10 the Day of the Month, I bring my Eye back to (17) in the left-hand Margent.

3. I look for 13 among the Primes, and from that guide my Eye to (1) in the Table, and finding the aforeſaid Number (17 in that Column, I do from that, guide my Eye to the right-hand Margent, and find that the Moon upon the Tenth of *May*, 1665. is entring into *Leo* ♌, and governs the Bowels and Belly.

A Table ſhewing the time of the Moons coming to the South, and quantity of her ſhining.

The Moons age.	Moons ſouthing & ſhin.	Moons age for her ſhi.
1 16	0 48	1 29
2 17	1 36	2 28
3 18	2 24	3 27
4 19	3 12	4 26
5 20	4 00	5 15
6 21	4 48	6 24
7 22	5 36	7 23
8 23	6 24	8 22
9 24	7 12	9 21
10 25	8 00	10 20
11 26	8 48	11 19
12 27	9 36	12 18
13 28	10 24	13 17
14 29	11 12	14 16
15 30	12 00	15

The uſe of this Table.

FInd the Moons age in the firſt Column, and next againſt the ſame towards the right hand, is the time of her coming to the South; which from the New Moon to the Full Moon, is always in the Afternoon, but from the Full to the New, it is in the Morning.

Example.

May 12. 1671. the Moon is fourteen days old, which I find in the firſt Column, againſt which, towards the right hand in the ſecond Column is 11. 12, which being before the Full of the Moon, I conclude that the Moon comes to ſouth *May* 12. 1671. at 11 a clock at Night, and 12 Minutes paſt.

To

To know how long the Moon Shineth.

Enter the third Column with the Moons age, and againſt it, on the left hand, you have the time of her ſhining, which all the time of her Encreaſe, being added to the hour of Sun riſing, gives the time of her riſing: But if added to the time of Sun ſetting, gives the time of her ſetting.

But after the Full.

Take the time of her ſhining from the Suns riſing, and it gives her riſing; and then take the ſame from the Sun ſetting, it gives the time of her ſetting.

Example.

May 12. 1671. the Moon is 14 days old, and I find 11 hours, 12 minutes, for the time of her ſhining, (which being added to the Suns riſing) upon the twelf of *May*, 1671. (*viz.* four hours) makes 3 of the clock, 12 minutes, for the time of the Moons riſing the next Morning.

Again, to the ſaid 11 hours, 12 minutes, add 8 hours from the Sun ſetting, it gives 7 hours, 12 minutes for the time of his ſetting.

Though theſe *Rules* are not altogether exact, yet they come near enough the truth, for ordinary uſe.

A Tide-Table of certain Havens in and about *England*, whereby may be known what *Moon* makes a *Full-Sea* in any of the ſaid places; and how many ho. and min. are to be added to the time of the Moons coming to the *South* for the time of *High-water*.

		H. M.
South and North	*Queenborough, Southamton, Portſmouth, Iſle of Wight, Spits, Kentiſh Knock*, half-Tide *at Dunkirk*	0 0
S by W N by E	*Rocheſter, Malden, Aberdeen, Redband*, Weſt-end of the *Nowr. Blacktail.*	0 45
S S W N N E	*Graveſend, Downs, Rumney, Tennet, Silly* halfe tide, *Blackneſs, Ramkines, Senehead.*	1 30
S W by S N E by N	*Dundee*, St. *Andrews, Lisborn*. Sr. *Lucas, Bell Iſle, Holy Iſle.*	2 15
S W N E	*London, Tinmouth, Hartlepool. Whitebay, Amſterdam, Gaſcoign, Britain. Galicia.*	3 0
S W by W N E by E	*Barwick, Flambrough-head, Bridlington bay, Burdeux, Oſtend, Fluſhing, Fountneſs.*	3 45
W S W E N E	*Scarborough* quarter-tide, *Lawrenas, Severn, Horkhave, Dungarum, Mounts-bay. Kingſale, Calice-Creek*	4 30
W by S E by N	*Newcaſtle, Humber, Falmouth Sally, Dartmouth, Torbay, St. Mallows, Foy, Garſy. Liz.*	5 15
East and Weſt	*Plimouth, Weymouth, Hull, Lyn, Davids head, Antwerp, Lundy, Holms of Briſtol.*	6 0
E by S W by N	*Briſtol, Foulneſs* at the Start.	6 45

		H. M.
E S E W N W	*Milford, Bridgewater, Lands-end, Waterford, Abermorick, Cape-Cleer, Texel.*	7 30
S E by E N W by W	*Portland, Peterport, Harflew,* the *Hague,* S. *Magnes.* ſouth, *Dublin, Lambay. Macknels Cape.*	8 15
S E N W	*Pool,* S. *Hellen, Caines, Orkney, Fair-Iſles, Kilden, Man-Iſle, Baſs-Iſlands.*	9 0
S E by S N W by N	*Needles, Laiſto, North & South Foreland.*	9 45
S S E N N W	*Yarmouth, Dover, Harwich,* S. *John de Luce, Calice* Road, *Bullein.*	10 30
S by E N by W	*Rye, Winchelſey, Goree, Thames, Rhodes.*	11 15

The Uſe of the Tide-Table.

Example.

MAy 12. 1671. I would know the Full Sea at *London.*

1. By the foregoing Rules I find the Moon comes to South at 11 of the clock, and 12 Minutes paſt, at Night.

There I ſeek for *London* in this Table, where I find that a S. W. or N. E. Moon makes a Full Sea, and on the right-hand I find 3 hours 0 minutes, which muſt be added to the Moons Southing. That is 3 hours, 0 minutes, added to 11 ho. 12 min. makes 2 a clock and 12 minutes the next Morning, for High Water at *London-Bridge.*

So that for any place and day, the hours and min. in the Table, are to be added to the Moons Southing, which gives the true time of High-water for that place and day.

A

The time of the Suns Rising and Setting

Days of the Month.	January		February		March	
	Sun riſes.	Sun ſets.	Sun riſes.	Sun ſets.	Sun riſes.	Sun ſets.
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	8 9	3 51	7 18	4 42	6 20	5 40
2	8 8	3 51	7 17	4 43	6 18	5 42
3	8 7	3 53	7 15	4 45	6 16	5 44
4	8 6	3 54	7 13	4 47	6 14	5 46
5	8 4	3 55	7 11	4 49	6 11	5 49
6	8 2	3 56	7 9	4 51	6 8	5 51
7	8 1	3 58	7 7	4 58	6 6	5 54
8	8 0	4 0	7 5	4 55	6 4	5 56
9	7 58	4 1	7 3	4 57	6 2	5 58
10	7 56	4 3	7 1	4 59	6 0	6 0
11	7 55	4 5	6 59	5 1	5 58	6 2
12	7 54	4 6	6 57	5 3	5 56	6 4
13	7 52	4 8	6 54	5 6	5 54	6 6
14	7 51	4 9	6 52	5 8	5 52	6 8
15	7 50	4 10	6 50	5 10	5 50	6 10
16	7 48	4 12	6 48	5 12	5 48	6 12
17	7 46	4 14	6 46	5 14	5 46	6 14
18	7 44	4 16	6 44	5 16	5 44	6 16
19	7 43	4 17	6 42	5 18	5 42	6 18
20	7 41	4 19	6 40	5 20	5 40	6 20
21	7 39	4 21	6 38	5 22	5 38	6 22
22	7 37	4 23	6 36	5 24	5 36	6 24
23	7 35	4 24	6 34	5 26	5 34	6 26
24	7 34	4 26	6 32	5 28	5 32	6 28
25	7 32	4 28	6 30	5 30	5 30	6 30
26	7 30	4 30	6 29	5 31	5 28	6 32
27	7 28	4 31	6 27	5 33	5 26	6 34
28	7 26	4 34	6 24	5 36	5 24	6 36
29	7 24	4 36	6 22		5 22	6 38
30	7 22	4 38			5 20	6 40
31	7 20	4 40			5 18	6 42

throughout the whole Year.

Days of the Month.	April. Sun riſes.	April. Sun ſets.	May. Sun riſes.	May. Sun ſets.	June. Sun riſes.	June. Sun ſets.
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	5 16	6 44	4 19	7 42	3 42	8 18
2	5 14	6 46	4 17	7 43	3 42	8 18
3	5 12	6 48	4 15	7 45	3 42	8 18
4	5 10	6 50	4 14	7 46	3 41	8 19
5	5 8	6 52	4 12	7 48	3 41	8 19
6	5 6	6 54	4 11	7 49	3 41	8 19
7	5 5	6 55	4 10	7 50	3 41	8 19
8	5 3	6 57	4 8	7 52	3 41	8 19
9	5 2	6 58	4 7	7 53	3 41	8 19
10	5 0	7 0	4 6	7 54	3 41	8 19
11	4 57	7 3	4 4	7 56	3 41	8 19
12	4 55	7 5	4 3	7 57	3 41	8 19
13	4 53	7 7	4 2	7 58	3 41	8 19
14	4 51	7 9	4 0	8 0	3 41	8 19
15	4 49	7 11	3 59	8 1	3 41	8 19
16	4 47	7 13	3 58	8 2	3 41	8 19
17	4 45	7 15	3 57	8 3	3 41	8 19
18	4 43	7 17	3 56	8 4	3 41	8 19
19	4 41	7 19	3 55	8 5	3 42	8 18
20	4 39	7 21	3 53	8 7	3 42	8 18
21	4 38	7 22	3 52	8 8	3 42	8 18
22	4 36	7 24	3 51	8 9	3 43	8 17
23	4 34	7 26	3 50	8 10	3 44	8 16
24	4 32	7 28	3 49	8 11	3 44	8 16
25	4 30	7 30	3 48	8 12	3 45	8 15
26	4 28	7 32	3 47	8 13	3 46	8 14
27	4 27	7 33	3 46	8 14	3 47	8 13
28	4 25	7 35	3 45	8 15	3 48	8 12
29	4 23	7 37	3 45	8 15	3 49	8 11
30	4 21	7 39	3 44	8 16	3 50	8 10
31			3 43	8 17		

The time of the Suns Rising and Setting

Days of the Month.	July. Sun rises.	July. Sun sets.	August. Sun rises.	August. Sun sets.	September Sun rises.	September Sun sets.
	H M	H M	H M	H M	H M	H M
1	3 51	8 9	4 35	7 26	5 35	6 25
2	3 51	8 8	4 36	7 24	5 37	6 23
3	3 52	8 8	4 38	7 22	5 39	6 21
4	3 53	8 7	4 40	7 20	5 41	6 19
5	3 54	8 6	4 41	7 19	5 43	6 17
6	3 56	8 4	4 43	7 17	5 45	6 15
7	3 57	8 3	4 45	7 15	5 47	6 13
8	3 58	8 2	4 47	7 13	5 49	6 11
9	3 59	8 1	4 48	7 12	5 51	6 9
10	4 0	8 0	4 50	7 10	5 53	6 7
11	4 2	7 58	4 52	7 8	5 56	6 4
12	4 3	7 57	4 54	7 6	5 58	6 2
13	4 4	7 56	4 56	7 4	6 0	6 0
14	4 6	7 54	4 58	7 2	6 2	5 58
15	4 7	7 53	5 0	7 0	6 4	5 56
16	4 8	7 52	5 2	6 58	6 6	5 54
17	4 10	7 50	5 4	6 56	6 9	5 51
18	4 11	7 49	5 6	6 54	1 11	5 49
19	4 12	7 48	5 8	6 52	6 13	5 47
20	4 14	7 46	5 10	6 50	6 15	5 45
21	4 15	7 45	5 12	6 48	6 17	5 43
22	4 17	7 43	5 14	6 46	6 19	5 41
23	4 18	7 42	5 16	6 44	6 21	5 39
24	4 20	7 40	5 18	6 42	6 23	5 37
25	4 22	7 38	5 20	6 40	6 25	5 35
26	4 23	7 37	5 22	6 38	6 27	5 33
27	4 25	7 35	5 24	6 36	6 29	5 31
28	4 27	7 33	5 26	6 34	6 31	5 29
29	4 28	7 32	5 28	6 32	6 33	5 27
30	4 30	7 30	5 30	6 30	6 35	5 25
31	4 32	7 28	5 32	6 28		

throughout the whole Year.

Days of the Month.	October. Sun riſes. H.M.	October. Sun ſets. H.M.	Novemb. Sun riſes. H.M.	Novemb. Sun ſets. H.M.	December. Sun riſes. H.M.	December. Sun ſets. H.M.
1	6 38	5 22	7 38	4 22	8 16	3 44
2	6 40	5 20	7 39	4 21	8 16	3 44
3	6 42	5 18	7 41	4 19	8 17	3 43
4	6 44	5 16	7 42	4 18	8 17	3 43
5	6 46	5 14	7 44	4 16	8 18	3 42
6	6 48	5 12	7 46	4 14	8 18	3 42
7	6 50	5 10	7 47	4 13	8 18	3 42
8	6 52	5 8	7 49	4 11	8 19	3 41
9	6 54	5 6	7 51	4 9	8 19	3 41
10	6 56	5 4	7 52	4 8	8 19	3 41
11	6 58	5 2	7 54	4 6	8 19	3 41
12	7 0	5 0	7 55	4 5	8 19	3 41
13	7 2	4 58	7 57	4 3	8 19	3 41
14	7 4	4 56	7 58	4 2	8 19	3 41
15	7 6	4 54	8 0	4 0	8 19	3 41
16	7 8	4 52	8 1	3 59	8 18	3 42
17	7 10	4 50	8 3	3 57	8 18	3 42
18	7 12	4 48	8 4	3 56	8 18	3 42
19	7 14	4 46	8 5	3 55	8 17	3 43
20	7 16	4 44	8 7	3 53	8 17	3 43
21	7 18	4 42	8 8	3 52	8 16	3 44
22	7 20	4 40	8 9	3 51	8 16	3 44
23	7 22	4 38	8 10	3 50	8 15	3 45
24	7 23	4 37	8 11	3 49	8 14	3 46
25	7 25	4 35	8 12	3 48	8 14	3 46
26	7 27	4 33	8 12	3 48	8 13	3 47
27	7 29	4 31	8 13	3 47	8 12	3 48
28	7 31	4 29	8 14	3 46	8 12	3 48
29	7 33	4 27	8 14	3 46	8 11	3 49
30	7 34	4 26	8 15	3 45	8 10	3 50
31	7 36	4 24			8 9	3 51

A Table ſhewing the length of the longeſt Artificial Day, in all places from the Equinoctial, to the Poles of the World.

Heig. Pole.	Long. day H.	M.	Heig. Pole.	Long. day H.	M.	Heig. Pole.	Longeſt day. D.	H.	M.
00	12	00	47	15	42	68	42	01	16
06	12	20	48	15	52	69	54	16	25
12	12	42	49	16	00	70	64	13	46
16	12	58	50	16	10	71	74	00	00
20	13	12	51	16	20	72	82	06	36
24	13	30	52	16	30	73	89	04	58
27	13	42	53	16	42	74	96	17	00
30	13	56	54	16	54	75	104	01	04
32	14	06	55	17	08	76	110	07	27
34	14	16	56	17	22	77	116	14	22
35	14	22	57	17	36	78	122	17	06
36	14	28	58	17	52	79	127	09	55
37	14	34	59	18	10	80	134	04	58
38	14	38	60	18	30	81	139	31	36
39	14	44	61	18	54	82	145	06	43
40	14	52	62	19	20	83	152	02	06
41	14	58	63	19	50	84	156	03	03
42	15	04	64	20	24	85	161	05	23
43	15	12	65	21	10	86	166	11	23
44	15	18	66	22	18	87	171	21	47
45	15	26	66½	24	00	88	176	05	29
46	15	34	67	24	Days	89	181	21	58
						90	187	06	39